Question 1 [20 Marks]

If the dynamics of a crude oil spreading system is described by the expression given in Equation (1), where d is the distance of spread, α and β are model constants, and the time-series data generated from the experiments carried out on the system are as given in Table 1, estimate the values of α and β,

(a) manually,

(b) with the aid of Microsoft Excel using the regression tool,

(c) with the aid of MATLAB using the regress command, and

(d) in each of the cases (a) to (c), estimate the values of the (i) correlation coefficient (R), (ii) square of the correlation coefficient (R2), and

(e) comment on the results obtained in (d).

$d = \alpha\beta^t$ (1)

Table 1. Time-series data of the system

t (hr)	d (m)
0	2
1	5
2	19
3	50
4	151
5	470
6	1435
7	4512
8	12936
9	41125
10	111021

NB: To use the data in MATLAB, they should be read from a Microsoft Excel file and not by copy and paste

SOLUTION

A. MANUAL SOLUTION

$d=\alpha\beta^t$

$\ln d = \ln\alpha + t*\ln\beta$

where

$\ln d = y$

$\ln\alpha$ = intersect = a_0

$\ln\beta$ = slope = a_1

$t = x$

hence, lnd = lnα + t*lnβ becomes:

$y = a_0 + a_1x$

$\sum y = a_0N + a_1 \sum x$

$\sum xy = a_0\sum x + a_1 \sum x^2$

t = x	D	Lnd = y	Lnd^2	tlnd	t^2
0	2	0.69315	0.48045	0.00000	0
1	5	1.60944	2.59029	1.60944	1
2	19	2.94444	8.66972	5.88888	4
3	50	3.91202	15.30392	11.73607	9
4	151	5.01728	25.17310	20.06912	16
5	470	6.15273	37.85612	30.76366	25
6	1435	7.26892	52.83720	43.61352	36
7	4512	8.41450	70.80374	58.90147	49
8	12936	9.46777	89.63866	75.74216	64
9	41125	10.62437	112.87727	95.61934	81
10	111021	11.61747	134.96572	116.17475	100
$\sum x$ = 55	$\sum d$ = 171726	$\sum y$ = 67.72209	$\sum y^2$ =551.19619	$\sum xy$ = 460.11840	$\sum x^2$ = 385

$67.722099 = a_0 11 + a_1 55$ …………1

$460.11840 = a_0 55 + a_1 385$………..2

From equation (1); $a_0 = (67.722099 - a_1 55) /11$

By substituting $a_0 = (67.722099 - a_1 55) /11$ into equation (2)

$460.11840 = ((67.722099 - a_1 55) /11) * 55 + a_1 385$

$460.11840 = 338.610495 - a_1 275 + a_1 385$

$121.507905 = 110 a_1$

$a_1 = 121.507905/110 = 1.104617$

$a_0 = (67.722099 - (1.104617)* 55) /11 = 0.633469$

therefore, the constants α and β are:

$\ln\alpha$ = intersect = a_0 = 0.633469

α = exp(0.633469) = 1.88414

$\ln\beta$ = slope = a_1 = 1.104617

β = exp(1.104617) = 3.01807

i. The correlation coefficient R is:

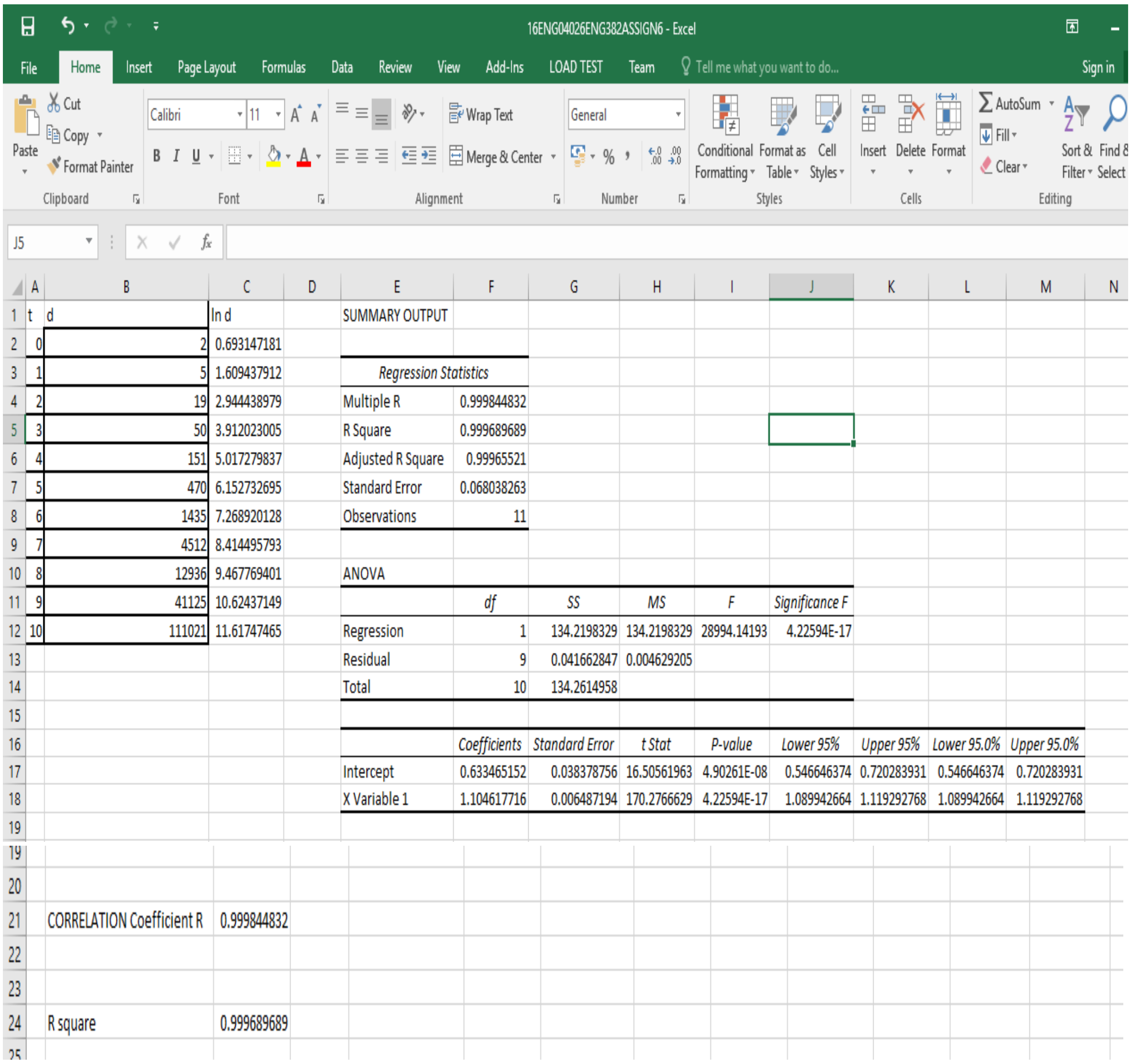
$$R = \frac{N\sum xy - (\sum x)(\sum y)}{\sqrt{((N\sum x^2 - (\sum x)^2)(N\sum y^2 - (\sum y)^2))}}$$

$$R = \frac{11*460.11840-(55)(67.72209)}{\sqrt{((11*385-55^2)(11*551.19619-67.72209^2))}} = 0.99985$$

ii. The square of the correlation coefficient R is:

$R^2 = (0.99985)^2 = 0.99970$

B. Solution with the aid of Microsoft Excel using the regression tool



t	d	ln d
0	2	0.693147181
1	5	1.609437912
2	19	2.944438979
3	50	3.912023005
4	151	5.017279837
5	470	6.152732695
6	1435	7.268920128
7	4512	8.414495793
8	12936	9.467769401
9	41125	10.62437149
10	111021	11.61747465

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.999844832
R Square	0.999689689
Adjusted R Square	0.99965521
Standard Error	0.068038263
Observations	11

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	134.2198329	134.2198329	28994.14193	4.22594E-17
Residual	9	0.041662847	0.004629205		
Total	10	134.2614958			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.633465152	0.038378756	16.50561963	4.90261E-08	0.546646374	0.720283931	0.546646374	0.720283931
X Variable 1	1.104617716	0.006487194	170.2766629	4.22594E-17	1.089942664	1.119292768	1.089942664	1.119292768

CORRELATION Coefficient R	0.999844832
R square	0.999689689

C. Solution with the aid of MATLAB using the regress command, and

MATLAB R2018a

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Command Window

INAIBO16ENG04026ENG382ASSIGN6.m

```
commandwindow
clear
clc
close all
format long g
Peter  = xlsread('ISAIAH');
[xr xc]= size(Peter);
x0 = ones(xr,1);
x1 = Peter(:,1);
y = Peter(:,3);
x = [x0 x1];
Result = regress(y,x)
a0 = Result(1);
a1 = Result(2);
ysim = a0+a1*x1;
Rvalue = corr(y,ysim)
Rsquare = Rvalue^2
```

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Command Window — Editor - INAIBO16ENG04026ENG382ASSIGN6.m

```
Result =

         0.633465152361569
          1.1046177162803


Rvalue =

          0.99984483235763


Rsquare =

         0.999689688792257

fx >>
```

(e) comment on the results obtained in (d).

The correlation coefficient (R) is 0.99984, meaning there is a strong positive correlation or relationship between the time t values and the distance of spread values.

Also, the R square value is 0.99969, meaning the model strongly related the data it is representing.