

1/1/ Solution

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x} \quad (1)$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad (2)$$

Solving equ. (1) & (2) simultaneously

Multiply equ. (1) by 6

$$\therefore 6 \frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x} \quad (3)$$

$$\frac{d^2y}{dx^2} = +16Ae^{-4x} + 36Be^{-6x} \quad (4)$$

$$6 \frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$$

$$\therefore A = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8e^{-4x}} \quad (5)$$

Substituting equ. (5) into equ. (1)

$$\frac{dy}{dx} = 4 \left(\frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{8e^{-4x}} \right) e^{-4x} - 6Be^{-6x}$$

$$\frac{dy}{dx} = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{2} - 6Be^{-6x}$$

Multiply through by 2

$$\therefore 2 \frac{dy}{dx} = \frac{6 \frac{dy}{dx}}{dx} + \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$2 \frac{dy}{dx} - \frac{6 \frac{dy}{dx}}{dx} = \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$-4 \frac{dy}{dx} - \frac{d^2y}{dx^2} = -12Be^{-6x}$$

$$-4 \frac{dy}{dx} - \frac{d^2y}{dx^2} = B$$

$$\therefore 4 \frac{dy}{dx} + \frac{d^2y}{dx^2} = B$$

Substitute A and B into the degenerate equation

$$y = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8e^{-4x}} + \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{12e^{-6x}}$$

$$y = \frac{6 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8} + \frac{4 \frac{dy}{dx} + \frac{d^2y}{dx^2}}{12}$$

$$y = \frac{-72 \frac{dy}{dx} - 12 \frac{d^2y}{dx^2} + 32 \frac{dy}{dx} + 8 \frac{d^2y}{dx^2}}{96}$$

$$y = \frac{-40 \frac{dy}{dx} - 4 \frac{d^2y}{dx^2}}{96}$$

$$96y = -40 \frac{dy}{dx} - 4 \frac{d^2y}{dx^2}$$

$$24y = -10 \frac{dy}{dx} - \frac{d^2y}{dx^2}$$

$$\therefore \frac{d^2y}{dx^2} + 10 \frac{dy}{dx} + 24y = 0$$