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Assignment 1

(1a) A differential equation is a relationship between an independent Variable 'x' and dependent Variable 'y' and one or more derivatives of y w.r.t x

Eg $dy/dx = y + y/x$

(b) $y = Ae^{-4x} + Be^{-6x}$

A Second order differential equation

This is because it contains two Variables

$y = Ae^{-4x} + Be^{-6x}$

Solution

$$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x} \quad --- (1)$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad --- (2)$$

Solving eqns (1) and (2) Simultaneously and multiplying eqn (1) by 6

$$6\frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x} \quad --- (3)$$

$$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \quad --- (4)$$

$$6\frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$$

$$A = 6\frac{dy}{dx} + \frac{d^2y}{dx^2} \quad --- (5)$$

Substitute eqn (5) into eqn (1)

$$\frac{dy}{dx} = 4\left(\frac{6\frac{dy}{dx} + d^2y/dx^2}{8e^{-4x}}\right)e^{-4x} - 6Be^{-6x}$$

$$\frac{dy}{dx} = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{2} - 6Be^{-6x}$$

Multiply through by 2

$$2\frac{dy}{dx} = 6\frac{dy}{dx} + \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$\frac{2dy}{dx} - \frac{6dy}{dx} = \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$-4\frac{dy}{dx} - \frac{d^2y}{dx^2} = -12Be^{-6x}$$

$$\frac{-4\frac{dy}{dx} - \frac{d^2y}{dx^2}}{-12e^{-6x}} = B$$

$$B = \frac{4\frac{dy}{dx} + \frac{d^2y}{dx^2}}{12e^{-6x}}$$

Substitute A and B into the degenerate equation

$$y = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8e^{-4x}} \times e^{-4x} + \frac{4\frac{dy}{dx} + \frac{d^2y}{dx^2}}{12e^{-6x}} \times e^{-6x}$$

$$y = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8} + \frac{4\frac{dy}{dx} + \frac{d^2y}{dx^2}}{12}$$

$$y = \frac{18\frac{dy}{dx} + 3\frac{d^2y}{dx^2} - 8\,dy/dx - 2\,d^2y/dx^2}{-24}$$

$$y = \frac{10\,dy/dx + d^2y/dx^2}{-24}$$

$$-24y = 10\,dy/dx + d^2y/dx^2$$

$$\frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 24y = 0$$