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Mechatronics Engineering
Assignment 4

Answers

Let $y(t)$ be the amount of air at any time t in ft^3 in the room.

$\frac{dy}{dt}$ = Fresh air inflow rate - Fresh air outflow rate

Fresh air inflow - $600 \text{ ft}^3/\text{min}$

Fresh air outflow - Note: The amount being out of the room is a function of the amount in room

$$\frac{600}{2000} = 0.03 \text{ min}^{-1}$$

i.e. 0.03 of $y(t)$ is the outflow $= 0.03y \text{ ft}^3/\text{min}$

Now,

$$\begin{aligned} \frac{dy}{dt} &= 600 - 0.03y \\ &= -0.03y + 600 \\ &= -0.03(y - 2000) \end{aligned}$$

This equation can be separated and integrated

$$\frac{d}{(y - 2000)} = -0.03 dt$$

find the integral of both sides

$$\ln(y - 2000) = -0.03t + C$$

$$y - 2000 = e^{C - 0.03t}$$

$$y - 2000 = e^{-0.03t} \cdot e^C$$

$$C = e^C = \text{initial equation}$$

$$y - 2000 = e^{-0.03t} \cdot C \quad \text{--- (1)}$$

At $t=0$, $y(0)=0$ since the room contained no fresh air initially
put $y=0$; $t=0$ in eqn (1)

$$0 - 2000 = e^{-0.03(0)} \cdot C$$

$$0 - 2000 = e^0 \cdot C$$

$$0 - 2000 = 1(C)$$

$$C = -2000 \quad \text{--- (2)}$$

Put eqn (2) in eqn (1)

$$y = 20000 - 20000 e^{-0.03t}$$
$$y = 20000 (1 - e^{-0.03t}) \quad (3)$$

Equation (3) above is the model for the amount of fresh air in the room.

Calculate the time at which 90% of the air in the room will become fresh

$$90\% = \frac{90}{100} = 0.9$$

$$y = 0.9 \times 20,000 \text{ i.e. } 90\% \text{ of air in the room.}$$
$$= 18000 \text{ ft}^3$$

$$y = 20000 (1 - e^{-0.03t})$$
$$18000 = 20000 (1 - e^{-0.03t})$$

$$0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 1 - 0.9$$

$$e^{-0.03t} = 0.1$$

$$-0.03t = \ln(0.1)$$

$$t = \frac{\ln(0.1)}{-0.03}$$

$$= \frac{-2.303}{-0.03}$$

$$= 76.77 \text{ mins} \approx 77 \text{ mins}$$

With the aid of MATLAB, plot the dynamic response of the amount of fresh air in the room for $T = 0$ to $T = 6 \text{ hrs}$ using a step time of 5 min

Note: $T = 6 \text{ hrs}$

$$= 6 \times 60$$

$$= 360 \text{ mins}$$

Solution

Command window

clear all

clc

close all

syms y,t

$$y = 20000 * (1 - \exp(0.03 * t))$$

$$t = 0.5 : 360$$

$y_n = \text{subs}(y)$

Plot (t, y_n)

X label (Time (min))

Y label (Flowrate of fresh air (ft^3/min))

Grid on.

Grid minor.

Axis tight.

Output