

ASSIGNMENT 4.  
Let  $y(t)$  be the amount of air in the room at any time  $t$ .

$$\frac{dy}{dt} = \text{fresh air in flow rate} - \text{fresh air outflow rate.}$$

$$\text{fresh air inflow} = 600 \text{ ft}^3/\text{min}$$

$$\text{fresh air outflow} = \frac{600}{20000} = 0.03 \text{ min}^{-1}$$

i.e. 0.03 of  $y(t)$  is the outflow

Now,

$$\begin{aligned} \frac{dy}{dt} &= 600 - 0.03y \\ &= -0.03y + 600 \\ &= -0.03(y - 20000) \end{aligned}$$

Thus the eqn is separable and can be written as

$$\frac{dy}{y - 20000} = -0.03 dt$$

Integrate both sides.

$$\ln(y - 20000) = -0.03t + c.$$

$$y - 20000 = e^{-0.03t + c}$$

$$y - 20000 = e^{-0.03t} e^c$$

Recall  $e^c = e^{\ln C} = C$  = initial condition

$$y - 20000 = e^{-0.03t} C \quad (*)$$

at  $t = 0$   $y(0) = 0$  Since the room contained no fresh air

$$\text{Initially } y = 20000 = (e^{-0.03t})$$

$$0 - 20000 = c$$

$$(t) = c = -20000 \quad \textcircled{a}$$

Put a in equation (\*)

$$y = 20000 - 20000 e^{-0.03t}$$

$$y = 20000 (1 - e^{-0.03t}) \quad **$$

The equation above is the model for the amount of fresh air in the room  
Calculate the time at which 90% of the air in the room will become fresh

$$90\% = \frac{90}{100}$$

$$= 0.9$$

$$y = 0.9 \text{ of } 20000$$

$$= 18000 \text{ ft}^2$$

$$y = 20000 (1 - e^{-0.03t})$$

$$18000 = 20000 (1 - e^{-0.03t})$$

$$0.9 = 1 - e^{-0.03t}$$

$$e^{-0.03t} = 1 - 0.9$$

$$e^{-0.03t} = 0.1$$

$$-0.03t = \ln(0.1)$$

$$t = \frac{\ln(0.1)}{-0.03}$$

$$-0.03$$

30:03:2019



$$t = \frac{-2303}{-0.03}$$

$$= 76.77$$

$$\approx 77 \text{ mins}$$

c) With the aid of matlab  
Plot a dynamic response.

N/B  $t = 6 \text{ hours} = 6 \times 60 = 360 \text{ minutes}$   
Codes.

Command window

clear all

clc

close all

Sym  $y, t, h$

$$y = 20000 * (1 - \exp(-0.03 * t))$$

$$t = 0:5:360$$

$$y_n = \text{subs}(y)$$

plot(t,  $y_n$ )

xlabel('Time(min)')

ylabel('FLOWRATE OF FRESH AIR (ft<sup>3</sup>/min)')

grid on.

grid minor

axis right

d) Determine the steady-state value of the amount of fresh air in the room

The steady-state value is  
20000 ft<sup>3</sup> / at 215 mins of  
exponential approach

e) Comment on answer

The function shows an exponential approach to the limit of  $20000 \text{ ft}^2$  as  $y$  increases with time. Also when the steady state value approach  $20000 \text{ ft}^3$  at  $45 \text{ mins}$  and continues till  $360$  minutes