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17/ENG06/014
MECHANICAL ENGINEERING
ENG 282 (Assignment 4)

a Let $y(t)$ be the amount of air at time t in (ft^3) in the room

$\frac{dy}{dt}$ = Fresh air inflow rate - Fresh air outflow rate

Fresh air inflow → $600 ft^3 min^{-1}$

Fresh air outflow → $\frac{600}{20000} = 0.03 min$

i.e 0.03 of $y(t)$ is the outflow
$= 0.03y\, ft^3$

$$\frac{dy}{dt} = 600 - 0.03y$$

$$= -0.03y + 600$$

$$= -0.03(y - 20000)$$

Thus, the equation is separable and can be solved

$$\frac{dy}{(y - 20000)} = 0.03dt$$

Integrate both sides

$$\ln(y - 20000) = -0.03t + c$$

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$$y - 20000 = e^{-0.03t + c}$$

$$y - 20000 = e^{-0.03t} e^{c}$$

Recall $C = e^{c}$ = Initial Condition

$$y - 20000 = e^{-0.03t} C \longrightarrow i$$

At $t = 0$, $y(t) = 0$ since the room contained no fresh air initially

$$y - 20000 = Ce^{-0.03(t)}$$

$$0 - 20000 = C$$

$$C = -20000 \longrightarrow ii$$

Put eqn ii in eqn i

$$y = 20000 - 20000e^{-0.03t}$$

$$y = 20000\left(1 - e^{0.03t}\right) \longrightarrow iii$$

The equation above is the model for the amount of fresh air in the room

b y $90\% = \frac{90}{100} = 0.9$

$y = 0.9$ of 20000

$= 0.9 \times 20000$

$= 18000\ Ft^3$

$$\therefore y = 20000\left(1 - e^{-0.03t}\right)$$

$$18000 = 20000\left(1 - e^{-0.03t}\right)$$

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$$0.9 = 1 - e^{-0.08t}$$
$$e^{-0.08t} = 1 - 0.9$$
$$e^{-0.08t} = 0.1$$
$$-0.08t = \ln(0.1)$$
$$t = \frac{\ln(0.1)}{-0.03}$$
$$t = \frac{-2303}{-0.03} = 76.77 \text{ mins} \simeq 77 \text{ mins}$$

c Commandwindow

```
Clear all
Clc
Close all
Sym.Y, t, k
Y = 20000 (1 - exp(-0.03^2 t))
t = 0:5:360
Yn = subs(Y)
Plot (t, Yn)
Xlabel ('TIME (min)')
ylabel ('FLOWRATE OF FRESH AIR (Ft^3/min)')
Grid on
Grid minor
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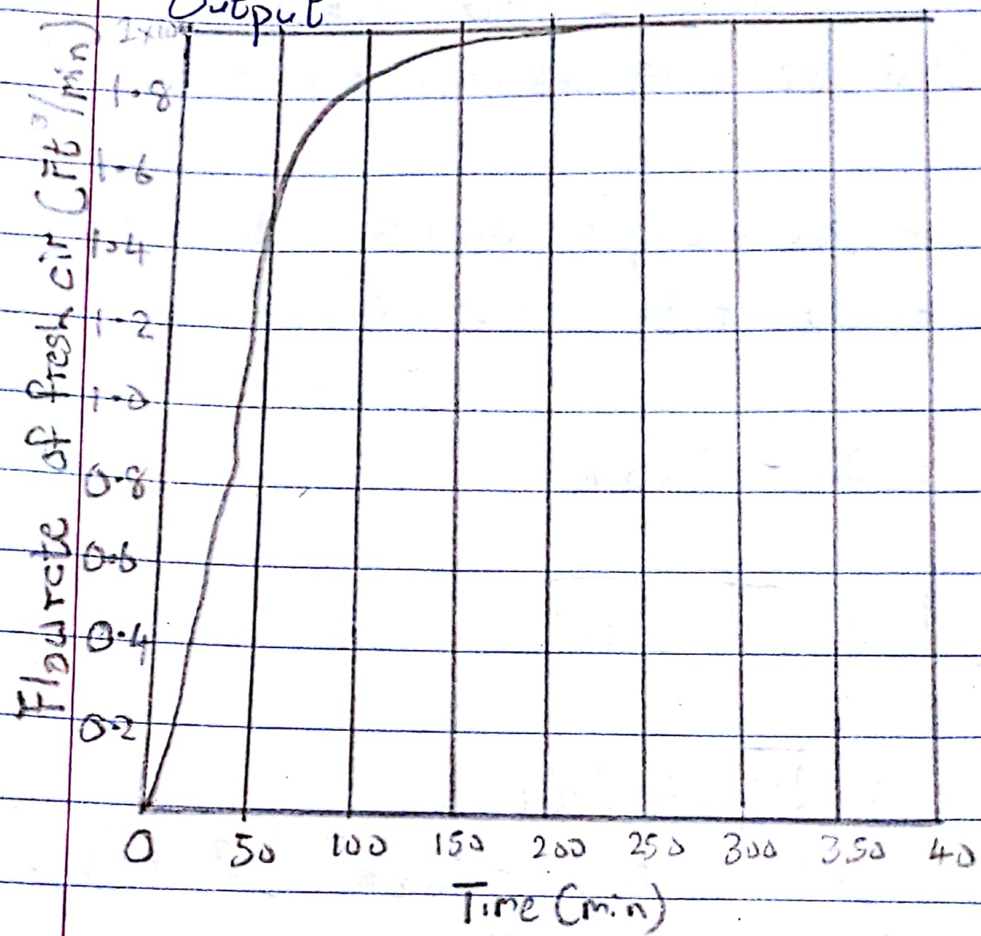
axis tight
Output



d The steady-state value is 20000 ft^3 at 215 mins of exponential approach

e The function shows an exponential approach to the limit of 20000 ft^3 as y increases with time, when the steady ~~stat~~ state value approach 2000 ft^3 at 215 mins and continues till 360 mins. The model becomes more realistic in Pneumatic technology.