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 Course: ENG 282

Assignment 1

(a) A differential equation is a relationship between an independent variable (x) and dependent variable (y) and one or more derivatives of y with respect to x

Examples: (i) $\frac{dy}{dx} = 2 + \frac{y}{x}$
 (ii) $\frac{dy}{dx} = y + \frac{y}{x}$

(b) $y = Ae^{-4x} + Be^{-6x}$

(i) A second order differential equation
 (ii) A second order differential equation can be formed because it contains 2 constants in the degenerate equation

(iii) $y = Ae^{-4x} + Be^{-6x}$

Solution

$\frac{dy}{dx} = -4Ae^{-4x} - 6Be^{-6x} \dots (1)$

$\frac{d^2y}{dx^2} = 16Ae^{-4x} + 36Be^{-6x} \dots (2)$

Solving equ (1) and (2) simultaneously

multiply equ (1) by 6

$6\frac{dy}{dx} = -24Ae^{-4x} - 36Be^{-6x} \dots (3)$

$\frac{d^2y}{dx^2} = +16Ae^{-4x} + 36Be^{-6x} \dots (4)$

$6\frac{dy}{dx} + \frac{d^2y}{dx^2} = -8Ae^{-4x}$

$\therefore A = \frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{-8e^{-4x}} \dots (5)$

Sub equ (5) into equ (1)

$\frac{dy}{dx} = 4 \left(\frac{6\frac{dy}{dx} + \frac{d^2y}{dx^2}}{8e^{-4x}} \right) e^{-4x} - 6Be^{-6x}$

$$\frac{dy}{dx} = 6 \frac{dy}{dx} + \frac{d^2y}{dx^2} - 6Be^{-6x}$$

Multiply through by 2

$$\therefore 2 \frac{dy}{dx} = 6 \frac{dy}{dx} + \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$2 \frac{dy}{dx} - 6 \frac{dy}{dx} = \frac{d^2y}{dx^2} - 12Be^{-6x}$$

$$-4 \frac{dy}{dx} - \frac{d^2y}{dx^2} = -12Be^{-6x}$$

$$-4 \frac{dy}{dx} - \frac{d^2y}{dx^2} = B$$

$$\therefore 4 \frac{dy}{dx} + \frac{d^2y}{dx^2} = B$$

Sub A and B into the degenerate equation

$$\therefore y = 6 \frac{dy}{dx} + \frac{d^2y}{dx^2} \times e^{-4x} + 4 \frac{dy}{dx} + \frac{d^2y}{dx^2} \times e^{-6x}$$

$$y = 6 \frac{dy}{dx} + \frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + \frac{d^2y}{dx^2}$$

$$y = -72 \frac{dy}{dx} - 12 \frac{d^2y}{dx^2} + 32 \frac{dy}{dx} + 8 \frac{d^2y}{dx^2}$$

$$y = -40 \frac{dy}{dx} - 4 \frac{d^2y}{dx^2}$$

$$96y = -40 \frac{dy}{dx} - 4 \frac{d^2y}{dx^2}$$

$$24y = -10 \frac{dy}{dx} - \frac{d^2y}{dx^2}$$

$$\therefore \frac{d^2y}{dx^2} + 10 \frac{dy}{dx} + 24y = 0$$