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MATRIC NO: 17/ENG02/061

DEPARTMENT : COMPUTER ENGINEERING

(a) Define mathematical modelling

Mathematical modelling is a mathematical representation of a system and the simulation of a system which involves solving the model and obtaining its output variable for different values of its input variable.

(b) Outline two methods of obtaining models for engineering systems

(i) By Differentiation

(ii) By use of the Balance Law

(c) A thermometer that initially reads 10°C is used to measure the temperature of a system. The temperature of the thermometer is discovered to be 20°C after 5 minutes of inserting it into the system. If the actual temperature of the system is 25°C,

(i) develop a model for the system

Applying Newton's Law Of Cooling

$$\frac{dT}{dt} = K(T - T_A)$$

where T is the temperature reading of the thermometer and

T_A is the actual temperature of the system

$$\frac{dT}{dt} = K(T - T_A) \quad \text{---} (1)$$

Initial temperature, $T_0 = 10°C$

$$dT = K(T - T_A)dt$$

$$\frac{dT}{T - T_A} = Kdt \quad \ldots (2)$$

Integrating eqn (2)

$$\int \frac{dT}{T - T_A} = \int Kdt$$

$\ln|T-T_A| = kt + C^*$

$T-T_A = e^{kt} \cdot e^{C^*}$

$T-T_A = e^{kt} \cdot C$ (since $C = e^{C^*}$)

$T_{(t)} = T_A + Ce^{kt}$

At the initial temperature, $T_{(0)} = 10°C$

$10 = 25 + Ce^{kt}$

$10 - 25 = Ce^{kt}$

$-15 = Ce^{kt}$

At $T_{(0)}$, $t = 0$

$\therefore -15 = Ce^{k(0)}$

$\therefore C = -15$ and $T_{(t)} = 25 - 15e^{kt}$

To determine k,

At a temperature, $T = 20°C$ and time, $t = 5$ minutes

$20 = 25 - 15e^{5k}$

$20 - 25 = -15e^{5k}$

$-5 = -15e^{5k}$

$e^{5k} = -5/-15$

$e^{5k} = 5/15$

$e^{5k} = 0.333$

$5k = \ln(0.333)$

$5k = -1.1087$

$k = -0.22$

$\therefore$ ~~At t = 5 minutes~~

$T_{(t)} = 25 - 15e^{-0.22t}$ is the particular solution for this system.

(iii) simulate the developed model for time t=0 to t=60 min using a step time of 1 min with the aid of Microsoft Excel

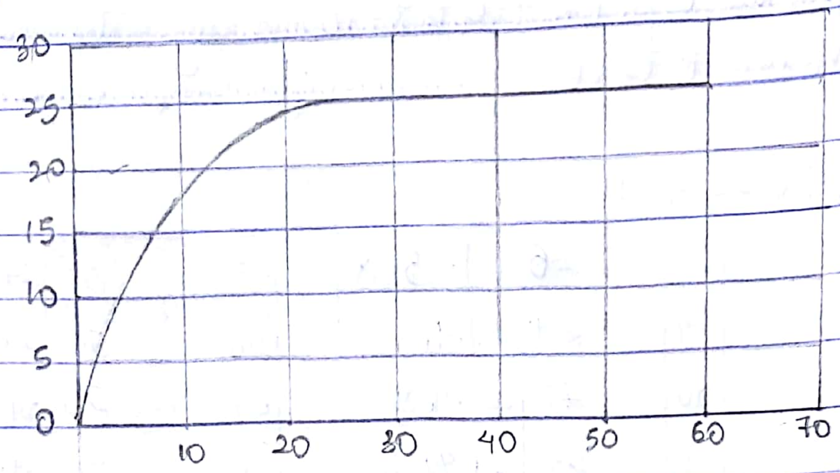
Output.

$fx = 25-15*EXP(-0.22*A2)$

1	t	T
2	0	10
3	1	12.96222
4	2	15.33945
5	3	17.24723
6	4	18.77826
7	5	20.00693
8	6	20.99297
9	7	21.78428
10	8	22.41933
11	9	22.92896
12	10	23.33795
13	11	23.66618
14	12	23.92958
15	13	24.14097
16	14	24.31061
17	15	24.44675
18	16	24.55601
19	17	24.64369
20	18	24.71405
21	19	24.77052
22	20	24.81584
23	21	24.85221
24	22	24.88139
25	23	24.90482
26	24	24.92361
27	25	24.9387
28	26	24.9508
29	27	24.9605
30	28	24.9683
31	29	24.9746
32	30	24.9796
33	31	24.9836
34	32	24.9869
35	33	24.9895
36	34	24.9915
37	35	24.9932
38	36	24.9945
39	37	24.9956
40	38	24.9965
41	39	24.9972
42	40	24.9977
43	41	24.9982
44	42	24.9985
45	43	24.9988
46	44	24.9991
47	45	24.9992
48	46	24.9994
49	47	24.9995
50	48	24.9996
51	49	24.9997
52	50	24.9997
53	51	24.9998
54	52	24.9998
55	53	24.9999
56	54	24.9999
57	55	24.9999
58	56	24.9999
59	57	24.9999
60	58	25
61	59	25
62	60	25

Scanned by CamScanner

Graph



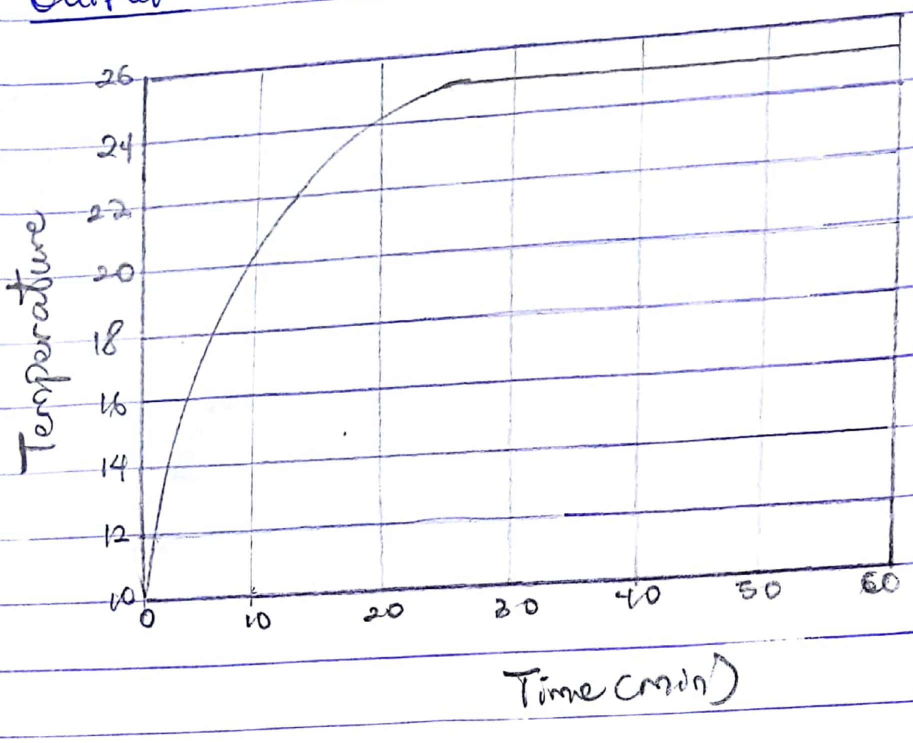
(iii) Obtain the dynamic response of the system with the aid of MATLAB, without using 'syms' command, for t = 0 to t = 60 min using a step of 1 min

SCRIPT

command window

```
clear
clc
close all
t = 0:1:60
T = 25-15*exp(-0.22*t)
Tm = subs(T)
plot(t,Tm)
grid on
grid minor
axis tight
xlabel ('Time (min)')
ylabel ('Temperature')
```

OUTPUT



(iv) Using either of the dynamic responses, write the steady-state temperature of the system

From the dynamic response obtained, the steady-state temperature of the system is 25°c at 24 minutes.

(v) Using the developed model equation, evaluate the temperature of the thermometer as $t \to \infty$

As $t \to \infty$, the temperature of the thermometer (T)

$\lim_{t \to \infty} T = ?$ To find the limiting value of T, trying for different values of t

when $t = 40$, $T = 25 - 15e^{-0.22(40)} = 24.998$

$t = 400$, $T = 25 - 15e^{-0.22(400)} = 25$

$t = 4000$, $T = 25 - 15e^{-0.22(4000)} = 25$

$t = 40000$, $T = 25 - 15e^{-0.22(40000)} = 25$

Therefore, $\lim_{t \to \infty} 25 - 15e^{-0.22t} = 25$

As time (t) increases without bound, the temperature of the system remains to be 25°c. Therefore, the temperature as $t \to \infty$ is 25°c.