a) $F(x) = e^{-0.5x}(4-x) - 2$.

Given initial guess value of $0.5 = x_0$

Maximum percentage absolute error $= 1E-9$

i) Find the root of the function.

Solution.

$f(x) = (4-x)e^{-0.5x} - 2$.

To find the root

When $x = 0$, $f(x) = (4-0)e^{-0.5} - 2 = 2$.

$f(x) = 2$.

When $x = 1$, $f(x) = (4-1)e^{0.5(1)} - 2 = 0.180408$.

$f(x) = -0.180$.

Therefore, to find $f'(x)$.

Expanding $f(x) = e^{-0.5x} - xe^{-0.5x} - 2$

OR.

$f(x) = e^{-0.5x}(4-x) - 2$

$f'(x) \Rightarrow$ differentiating using product rule :

$$f(x) = \frac{d}{dx}\left[e^{-0.5x}(4-x)\right] - \frac{d(2)}{dx}$$

$$= \cancel{e^{-0.5x}} \cdot \cancel{\frac{d}{dx}}\left[e^{-0.5x}\cancel{(4-x)}\right.$$

$$= e^{-0.5x} \cdot \frac{d}{dx}(4-x) + (4-x) \cdot \frac{d}{dx}(e^{-0.5x}) - 0.$$

$$= e^{-0.5x} - 1 + (4-x) \cdot \frac{d}{dx}(e^{-0.5x}) - 0.$$

$$= -e^{-0.5x} - 1 + (4-x) - 0.5e^{-0.5x}.$$

$$= 4 \cdot -0.5e^{-0.5x} + x \cdot 0.5e^{-0.5x} - e^{-0.5x}.$$

$$= x \cdot 0.5e^{-0.5x} - 2e^{-0.5x} - e^{-0.5x}.$$

$$f'(x) = 0.5xe^{-0.5x} - 3e^{-0.5x}.$$

$f'(x) = e^{-0.5x}[0.5x - 3]$.

using Newton Raphson Method.

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}.$$

For percentage absolute error, $= \left[\frac{x_{k+1} - x_k}{x_{k+1}}\right] \times 100\%$

For iter 1;

Let $x_k = 0.5 = x_0$

$f(x_0) = (4 - 0.5)e^{-0.5(0.5)} - 2$.

$= 0.7258027407$.

$f'(x_0) = e^{-0.5(0.5)}[0.5 \times 0.5 - 3]$.

$= 2.141702153 = -2.141702153$.

$\therefore x_{k+1} = 0.5 - \frac{0.7258027407}{-2.141702153}$.

$x_{k+1} = 0.838890606$.

% absolute error $= \left[\frac{0.838890606 - 0.5}{0.838890606}\right] \times 100\%$.

% absolute error $= 40.397472999\%$.

iter 2:

Let $x_k = 0.838890606 = x_1$

$f(x_1) = (4 - 0.838890606)e^{-0.5(0.838890606)} - 2$.

$f(x_1) = 0.0781492 9794$.

$f'(x_1) = e^{-0.5 \times 0.838890606}[0.5 \times 0.838890606 - 3]$

$f'(x_1) = -1.696486032$.

$\therefore x_{k+1} = 0.838890606 - \left[\frac{0.0781492 9794}{-1.696486032}\right]$.

$x_{k+1} = 0.8849560003$.

Sumarry of obtained Results.

Table.

i	x_k	$f(x_k)$	$f'(x_k)$	x_{k+1}	% abs. erro
1	0.5	0.7258027407	-2.141702153	0.8388890606	40.3974729
2	0.8849560003	0.07814929794	-1.696486032	0.8849560003	5.2053880
3	0.885708605	0.0012365751	-1.643060762	0.885708605.	0.084972089
4	3.235214111×10^{-7}	3.235214111×10^{-7}	-1.042200929	0.885708862.	$2.22426113 7\times$
5	7.851×10^{-12}.	7.851×10^{-12}	-1.642200704	0.885708862	0