

Maths assignment 5

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(5)

a) Mathematical modelling is a process of setting up models, solving them mathematically and interpreting the results in physical and other terms.

b) Methods of Obtaining a model.

- By use of Torricelli's law e.g. leaking tank, outflow of water through a hole.
- By Newton's law of cooling e.g. heating an office building.

c) Ship.

$$T(0) = 10^{\circ}\text{C}, \quad T_{\infty} = 25^{\circ}\text{C}$$

$$T(5) = 20^{\circ}\text{C}$$

$$\frac{dT}{dt} = k(T - T_{\infty}) \quad \therefore dT = k(T - T_{\infty})dt$$

$$\int \frac{dT}{T - T_{\infty}} = \int k dt$$

$$= \ln(T - T_{\infty}) = kt + C$$

$$T - T_{\infty} = e^{kt + C}$$

$$T - T_{\infty} = e^{kt} + e^C$$

$$T = Ae^{kt} + T_{\infty}$$

$$\text{When } t = 0:$$

$$10 = A + 25$$

$$A = 10 - 25 = -15$$

$$T = 25 - 15e^{kt}$$

$$\text{at } t(5) = 20:$$

$$20 = 25 - 15e^{k(5)}$$

$$20 = 25 - 15e^{5k}$$

$$15e^{5k} = 25 - 20 = 5$$

$$15e^{5k} = 5$$

$$15e^{5k} = 5$$

$$e^{5k} = 0.3333$$

$$5k = \ln 0.3333$$

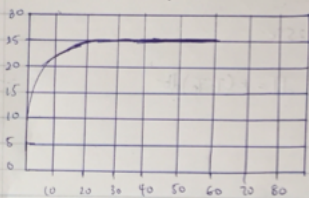
$$5k = -1.0986$$

$$k = -0.22$$

$$T(t) = 25 - 15e^{-0.22t}$$

b) Using excel

Using the equation $T(t) = 25 - 15e^{-0.22t}$ and at time 0 : 1 : 60 and graph of Temperature ($^{\circ}\text{C}$) against Time (t)



c) Using Matlab

Command window

```
clear
```

```
clc
```

```
close all
```

```
t = 0 : 1 : 60
```

```
T = 25 - 15 * exp(-0.22 * t)
```

```
Plot (t, T)
```

```
grid on
```

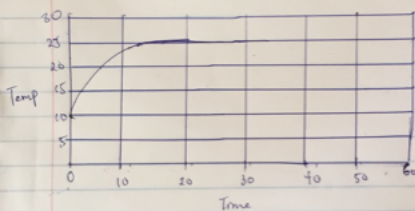
```
grid minor
```

```
xlabel ('Time (secs)')
```

```
ylabel ('Temperature (°C)')
```

```
grid on
```

```
grid minor
```



d) The steady state value of the system is at 25°C at 20mins.

e) Using the model eqn, the temperature of the thermometer at $t = 20$ is 25°C .

