ASSIGNMENT VII

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MECHANICAL ENGINEERING.

ENG 382: ENGINEERING MATHEMATICS IV.

Question

The model for the temperature distribution in a rod of length L = 6m is as given in eqn (1)

$$\frac{\partial T(x,t)}{\partial t} = c \frac{\partial^2 T(x,t)}{\partial x^2}$$

where

$$c = 2.2 \frac{cm^2}{hr}$$

with the conditions that the temp. (°C):

$$T(x,0) = 3x^2$$

$$T(0,t) = 0$$

$$T(L,t) = 100$$

Using $\Delta t = 0.02hr$ and $\Delta x = 0.3cm$, obtain the temp. profile of the system in $0 \leq t \leq 0.3hr$

(a) manually, in tabular form, solving up to $t = 0.02hr$ & $x = 6cm$,

(b) with the aid of Microsoft Excel, in tabular & 3D graphical forms and

(c) with the aid of MATLAB, in tabular & 3D graphical forms.

Solution

(a) Using Explicit forward Euler method.

$$u_i^{k+1} = r u_{i-1}^k + (1-2r) u_i^k + r u_{i+1}^k$$

$$r = c\frac{(\Delta t)}{(\Delta x)^2} = \frac{2.2 \times 0.02}{0.3^2}$$

$$r = 0.49$$

$$u_i^{k+1} = 0.49 u_{i-1,k} + 0.02 u_{i,k} + 0.49 u_{i+1,k}$$

For i = 1 to 19

1 → $u_1^{k+1} = 0.49 u_{0,k} + 0.02 u_{1,k} + 0.49 u_{2,k}$

2 → $u_2^{k+1} = 0.49 u_{1,k} + 0.02 u_{2,k} + 0.49 u_{3,k}$

3 → $u_3^{k+1} = 0.49 u_{2,k} + 0.02 u_{3,k} + 0.49 u_{4,k}$

4 → $u_4^{k+1} = 0.49 u_{3,k} + 0.02 u_{4,k} + 0.49 u_{5,k}$

5 → $u_5^{k+1} = 0.49 u_{4,k} + 0.02 u_{5,k} + 0.49 u_{6,k}$

6 → $u_6^{k+1} = 0.49 u_{5,k} + 0.02 u_{6,k} + 0.49 u_{7,k}$

7 → $u_7^{k+1} = 0.49 u_{6,k} + 0.02 u_{7,k} + 0.49 u_{8,k}$

8 → $u_8^{k+1} = 0.49 u_{7,k} + 0.02 u_{8,k} + 0.49 u_{9,k}$

9 → $u_9^{k+1} = 0.49 u_{8,k} + 0.02 u_{9,k} + 0.49 u_{10,k}$

10 → $u_{10}^{k+1} = 0.49 u_{9,k} + 0.02 u_{10,k} + 0.49 u_{11,k}$

11 → $u_{11}^{k+1} = 0.49 u_{10,k} + 0.02 u_{11,k} + 0.49 u_{12,k}$

12 → $u_{12}^{k+1} = 0.49 u_{11,k} + 0.02 u_{12,k} + 0.49 u_{13,k}$

13 → $u_{13}^{k+1} = 0.49 u_{12,k} + 0.02 u_{13,k} + 0.49 u_{14,k}$

14 → $u_{14}^{k+1} = 0.49 u_{13,k} + 0.02 u_{14,k} + 0.49 u_{15,k}$

15 → $u_{15}^{k+1} = 0.49 u_{14,k} + 0.02 u_{15,k} + 0.49 u_{16,k}$

16 → $U_{16}^{k+1} = 0.49\,U_{15,k} + 0.02\,U_{16,k} + 0.49\,U_{17,k}$

17 → $U_{17}^{k+1} = 0.49\,U_{16,k} + 0.02\,U_{17,k} + 0.49\,U_{18,k}$

18 → $U_{18}^{k+1} = 0.49\,U_{17,k} + 0.02\,U_{18,k} + 0.49\,U_{19,k}$

19 → $U_{19}^{k+1} = 0.49\,U_{18,k} + 0.02\,U_{19,k} + 0.49\,U_{20,k}$

Boundary Condition

$T(x,0) = 3x^2$; $0 < 0.36$ cm

$T(x_1,0) = 3(0.3)^2 = 0.27$

$T(x_2,0) = 3(0.6)^2 = 1.08$

$T(x_3,0) = 3(0.9)^2 = 2.43$

$T(x_4,0) = 3(1.2)^2 = 4.32$

$T(x_5,0) = 3(1.5)^2 = 6.75$

$T(x_6,0) = 3(1.8)^2 = 9.72$

$T(x_7,0) = 3(2.4)^2 = 13.23$

$T(x_8,0) = 3(2.4)^2 = 17.28$

$T(x_9,0) = 3(2.7)^2 = 21.87$

$T(x_{10},0) = 3(3)^2 = 27$

$T(x_{11},0) = 3(3.3)^2 = 32.67$

$T(x_{12},0) = 3(3.6)^2 = 38.88$

$T(x_{13},0) = 3(3.9)^2 = 45.63$

$T(x_{14},0) = 3(4.2)^2 = 52.92$

$T(x_{15},0) = 3(4.5)^2 = 60.75$

$T(x_{16},0) = 3(4.8)^2 = 69.12$

$T(x_{17},0) = 3(5.1)^2 = 78.03$

$T(x_{18},0) = 3(5.4)^2 = 87.48$

$T(x_{19},0) = 3(5.7)^2 = 94.74$

$T(x_{20},0) = 3(6)^2 = 108$

$T(0,t) = 0$,

$T(L,t) = 108$

Temperature has a range of 0 to 0.3 hr with step size of 0.02 hr.

For 0.02hr , K=0

$T_{1,1} = 0.49 u_{0,0} + 0.49 u_{2,0} + 0.02 u_{1,0}$

$= 0.49(0) + 0.49(1.08) + 0.02(0.27) = 0.5346$

$T_{2,1} = 0.49 u_{1,0} + 0.49 u_{3,0} + 0.02 u_{2,0}$

$= 0.49(0.27) + 0.49(2.43) + 0.02(1.08) = 1.3446$

$T_{3,1} = 0.49 u_{2,0} + 0.49 u_{4,0} + 0.02 u_{3,0}$

$= 0.49(1.08) + 0.49(4.32) + 0.02(2.43) = 2.6946$

$T_{4,1} = 0.49 u_{3,0} + 0.49 u_{5,0} + 0.02 u_{4,0}$

$= 0.49(2.43) + 0.49(6.75) + 0.02(4.32) = 4.5846$

$T_{5,1} = 0.49 u_{4,0} + 0.49 u_{6,0} + 0.02 u_{5,0}$

$= 0.49(4.32) + 0.49(9.72) + 0.02(6.75) = 7.0146$

$T_{6,1} = 0.49 u_{5,0} + 0.49 u_{7,0} + 0.02 u_{6,0}$

$= 0.49(6.75) + 0.49(13.23) + 0.02(9.72) = 9.9846$

$T_{7,1} = 0.49 u_{6,0} + 0.49 u_{8,0} + 0.02 u_{7,0}$

$= 0.49(9.72) + 0.49(17.25) + 0.02(13.23) = 13.4946$

$T_{8,1} = 0.49 u_{7,0} + 0.49 u_{9,0} + 0.02 u_{8,0}$

$= 0.49(13.28) + 0.49(21.87) + 0.02(17.28) = 17.5446$

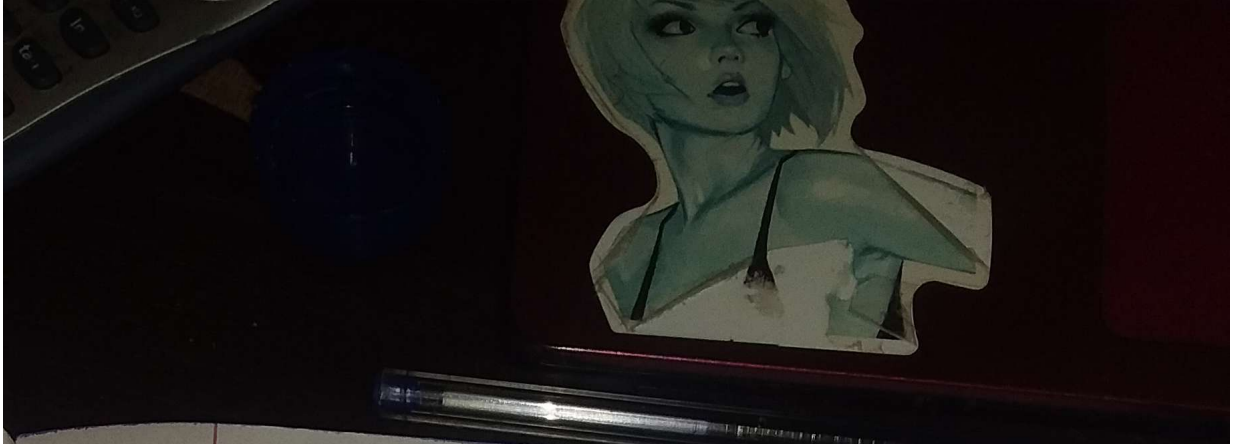
$T_{9,1} = 0.49 u_{8,0} + 0.49 u_{10,0} + 0.02 u_{9,0}$

$= 0.49(17.28) + 0.49(27) + 0.02(21.87) = 22.1346$

$T_{10,1} = 0.49 u_{9,0} + 0.49 u_{11,0} + 0.02 u_{10,0}$

$= 0.4(21.87) + 0.49(32.67) + 0.02(27)$

$= 27.2646$



$T_{11,1} = 0.49\,U_{10,0} + 0.49\,U_{12,0} + 0.02\,U_{11,0}$

$= 0.49(27) + 0.49(38.88) + 0.02(32.67) = 32.9346$

$T_{12,1} = 0.49\,U_{11,0} + 0.49\,U_{13,0} + 0.02\,U_{12,0}$

$= 0.49(32.67) + 0.49(45.63) + 0.02(38.88) = 39.1446$

$T_{13,1} = 0.49\,U_{12,0} + 0.49\,U_{14,0} + 0.02\,U_{13,0}$

$= 0.49(38.88) + 0.49(59.2) + 0.02(45.63) = 45.8946$

$T_{14,1} = 0.49\,U_{13,0} + 0.49\,U_{15,0} + 0.02\,U_{14,0}$

$= 0.49(45.63) + 0.49(60.75) + 0.02(52.92) = 53.1846$

$T_{15,1} = 0.49(U_{14,0} + 0.49\,U_{16,0} + 0.02\,U_{15,0}$

$= 0.49(52.92) + 0.49(69.12) + 0.02(60.75) = 61.0146$

$T_{16,1} = 0.49\,U_{15,0} + 0.49\,U_{17,0} + 0.02\,U_{16,0}$

$= 0.49(60.75) + 0.49(78.03) + 0.02(69.12) = 69.3846$

$T_{17,1} = 0.49\,U_{16,0} + 0.49\,U_{18,0} + 0.02\,U_{17,0}$

$= 0.49(69.12) + 0.49(87.48) + 0.02(78.03) = 78.2946$

$T_{18,1} = 0.49\,U_{17,0} + 0.49\,U_{19,0} + 0.02\,U_{18,0}$

$= 0.49(78.03) + 0.49(97.49) + 0.02(87.48) = 87.7446$

$T_{19,1} = 0.49\,U_{18,0} + 0.49\,U_{20,0} + 0.02\,U_{19,0}$

$= 0.49(87.48) + 0.49(108) + 0.02(97.49)$

$= 97.7346$

Table for solving up to $t = 0.02hr$ & $x = 6cm$

x	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0
$T(0)$	0	0.27	1.08	2.43	4.32	6.75	9.72	13.23	17.28	21.87	27.00
0.02	0	0.5346	1.3446	2.6946	4.5846	7.0146	9.78461	13.4946	17.5446	22.1346	27.264

3.3	3.6	3.9	4.2	4.5	4.8	5.1	5.4	5.7
32.67	38.88	45.63	52.92	60.75	69.12	78.03	87.48	97.47
32.9346	39.1446	45.8946	53.1846	61.0146	69.3846	78.2946	87.7446	97.7346

6.0
108
108