

Maths 5

LAURENCE - ARIENNE DE A...
MECHANICS / 1002
MECHANICAL ENGINEERING
LEAD 222

1) Mathematical modelling is a process of setting up models, solving them mathematically and integrating the results in physical and other terms.

2) Methods of obtaining a model

By use of Torricelli's law e.g. leaking tank, outflow of water through a hole.

By Newton's Law of cooling e.g. heating an object, cooling

d) Solution

$$T_{\infty} = 10^{\circ}\text{C}, T_0 = 25^{\circ}\text{C}$$

$$T_{10} = 20^{\circ}\text{C}$$

$$\frac{dT}{dt} = k(T - T_{\infty}) \quad \therefore dT = k(T - T_{\infty})dt$$

$$\int_{T_0}^{T} \frac{dT}{T - T_{\infty}} = \int_0^t k dt$$

$$= \ln(T - T_{\infty}) = kt + C$$

$$T - T_{\infty} = e^{kt + C}$$

$$T - T_{\infty} = e^{kt} + e^C$$

$$T = Ae^{kt} + T_{\infty}$$

$$\text{when } t = 0$$

$$10 = A + 25$$

$$A = 10 - 25 = -15$$

$$T = 25 - 15e^{kt}$$

$$\text{at } t(10) = 20$$

$$20 = 25 - 15e^{k(10)}$$

$$15e^{10k} = 25 - 20 = 5$$

$$15e^{10k} = 5$$

$$15e^{10k} = 5$$

$$e^{10k} = \frac{1}{3} = 0.3333$$

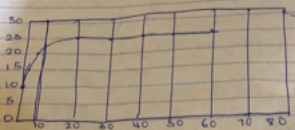
$$10k = \ln 0.3333$$

$$10k = -1.0986$$

$$k = -0.10986$$

$$T(t) = 25 - 15e^{-0.10986t}$$

b) Using excel
Using the equation $C(t) = 25 - 15e^{-0.22t}$ & t and at time
0:1:0 and graph of temperature (C) against time
(s)



c) Using Matlab

Command Window

clear

clc

close all

E = 0:1:60

T = 25 - 15 * exp(-0.22 * t)

Plot (t, T)

grid on

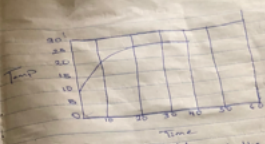
grid minor

xlabel ('Time (secs)')

ylabel ('Temperature (C)')

grid on

grid minor



- d) The Steady State Value of the System is at 25°C at 20min
- e) Using the Modeling equation, the temperature of the thermometer at $t=20$ is 25°C