

(1) Mathematical modelling is a mathematical representation of a system and simulation of a system which involves solving the model and obtaining its output variable for different values of its input variable or as input variable is changed from one value to another.

(2) Use of balance law
Differentiation.

e) $T_{at 0s} = 10^\circ C$
 $T_{at 5s} = 20^\circ C$
 Let $T_a = 25^\circ C$.

$$\frac{dT}{dt} = k(T - T_a)$$

$$k dt = \frac{dT}{T - T_a}$$

$$\int k dt = \int \frac{1}{T - T_a} dT$$

$$kt + c = \ln(T - T_a)$$

$$T - T_a = e^{kt + c}$$

$$T - T_a = e^{kt} \cdot e^c$$

$$\text{Let } T_0 = e^c$$

$$T - T_a = T_0 e^{kt}$$

$$T = T_a + T_0 e^{kt}$$

where $T = 10$

$$10 = T_0 e^{k(0)} + 25 \quad \text{at } t = 0$$

$$10 = T_0 + 25$$

$$T_0 = 10 - 25$$

$$T_0 = -15$$

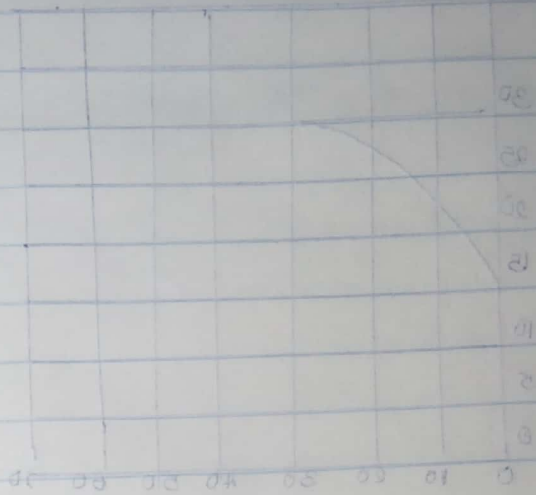
$$T = 25 - 15e^{kt}$$

At time $t = 5s$ and $T = 20^\circ C$

$$20 = 25 - 15e^{5k}$$

$$20 - 25 = -15e^{5k}$$

$$15e^{5k} = 5$$



substituted
 $10 = 25 - 15e^{5k}$
 $15e^{5k} = 15$
 $e^{5k} = 1$
 $5k = 0$
 $k = 0$
 $T = 25 - 15e^{0t} = 25 - 15 = 10$
 (put) $t = 5$
 $T = 25 - 15e^{5k}$
 $20 = 25 - 15e^{5k}$
 $15e^{5k} = 5$

$$e^{5k} = \frac{5}{15}$$

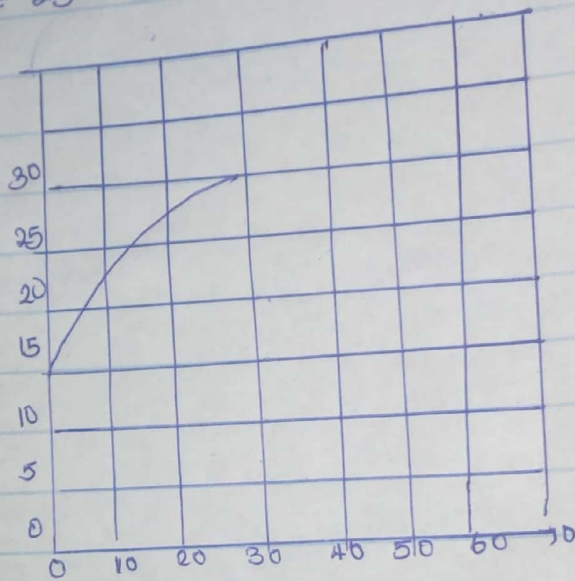
$$e^{5k} = 0.333$$

$$5k = \ln 0.333$$

$$5k = -0.0986$$

$$k = -0.02$$

$$T(t) = 25 - 15e^{-0.02t}$$



Command window

clear

clc

close all

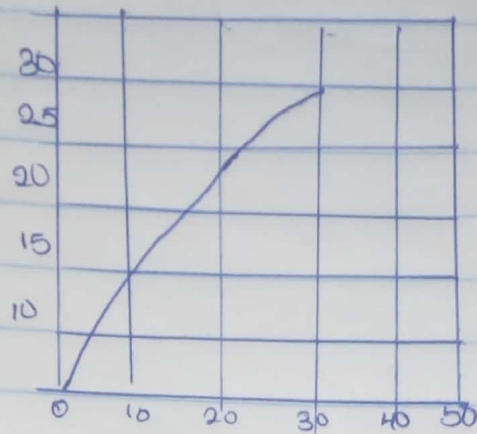
t=0:1:6

y=25-15*exp(-0.02*t)

plot(t,y)

grid on

grid minor



- (iv) Using ms excel's dynamic response, the steady state temperature of the system would be, 25° at 20 seconds.
- (v) Using the developed equation, the temperature of the thermometer as $t \rightarrow \infty = 25^{\circ}\text{C}$