

DEEMUA JOSEPH BRIGHT
16/ENG08/021

MECHANICAL ENGINEERING
ENG 382

ASSIGNMENT VII

1. The model for the temperature distribution in a rod of length $L = 6\text{cm}$ is as given in eqn 1.

$$\frac{dT(x,t)}{dt} = C \frac{d^2T(x,t)}{dx^2}$$

where $C = 2.2\text{cm}^2/\text{hr}$

With the conditions and the temperature ($^{\circ}\text{C}$)

$$T(x,0) = 3x^2$$

$$T(0,t) = 0$$

$$T(L,t) = -108$$

Using $\Delta t = 0.02\text{hr}$ and $\Delta x = 0.3\text{cm}$, obtain the temperature profile of the system for $0 \leq t \leq 0.3\text{hr}$.

- a. Manually, in tabular form solving up to $t = 0.02\text{hr}$ and $x = 6\text{cm}$

Solution

Using Explicit forward Euler Method

$$U_i^{k+1} = \gamma [U_{i+1}^k + U_{i-1}^k] + [1 - 2\gamma] U_i^k$$

$$\text{When } i = 1$$

$$U_{1,j+1} = \gamma U_{2,j} + \gamma U_{0,j} + [1 - 2\gamma] U_{1,j}$$

$$\text{but } \gamma = \frac{C \cdot \Delta t}{(\Delta x)^2} = \frac{2.2 \times 0.02}{(0.3)^2} = 0.49$$

$$[1 - (2 \times 0.49)] = 0.02$$

Rewriting the explicit forward euler method

for when $i = 1$ to 19

$$U_{1,j+1} = 0.49 U_{0,j} + 0.49 U_{2,j} + 0.02 U_{1,j}$$

$$U_{2,j+1} = 0.49 U_{1,j} + 0.49 U_{3,j} + 0.02 U_{2,j}$$

$T(0, t) = 0$, $T(L, t) = 108$
 temperature has a range of 0 to 0.3hr with step
 of 0.02hr. To get to 0.02hr, $j = 0$

When $j = 0$ (replacing U with T)

$$\begin{aligned} T_{1,1} &= 0.49U_{0,0} + 0.49U_{2,0} + 0.02U_{1,0} \\ &= 0.49(0) + 0.49(1.08) + 0.02(0.27) \\ &= 0.5346 \end{aligned}$$

$$\begin{aligned} T_{2,1} &= 0.49U_{1,0} + 0.49U_{3,0} + 0.02U_{2,0} \\ &= 0.49(0.27) + 0.49(2.43) + 0.02(1.08) \\ &= 1.3446 \end{aligned}$$

$$\begin{aligned} T_{3,1} &= 0.49U_{2,0} + 0.49U_{4,0} + 0.02U_{3,0} \\ &= 0.49(1.08) + 0.49(4.32) + 0.02(2.43) \\ &= 2.6946 \end{aligned}$$

$$\begin{aligned} T_{4,1} &= 0.49U_{3,0} + 0.49U_{5,0} + 0.02U_{4,0} \\ &= 0.49[2.43] + 0.49[6.75] + 0.02[4.32] \\ &= 4.5846 \end{aligned}$$

$$\begin{aligned} T_{5,1} &= 0.49U_{4,0} + 0.49U_{6,0} + 0.02U_{5,0} \\ &= 0.49[4.32] + 0.49[9.72] + 0.02[6.75] \\ &= 7.0146 \end{aligned}$$

$$\begin{aligned} T_{6,1} &= 0.49T_{5,0} + 0.49U_{7,0} + 0.02T_{6,0} \\ &= 0.49[6.75] + 0.49[13.23] + 0.02[9.72] \\ &= 9.9846 \end{aligned}$$

$$\begin{aligned} T_{7,1} &= 0.49T_{6,0} + 0.49T_{8,0} + 0.02T_{7,0} \\ &= 0.49[9.72] + 0.49[17.25] + 0.02[13.23] \\ &= 13.4946 \end{aligned}$$

$$\begin{aligned} T_{8,1} &= 0.49T_{7,0} + 0.49T_{9,0} + 0.02T_{8,0} \\ &= 0.49[13.23] + 0.49[21.87] + 0.02[17.23] \\ &= 17.5446 \end{aligned}$$

$$\begin{aligned} T_{9,1} &= 0.49T_{8,0} + 0.49T_{10,0} + 0.02T_{9,0} \\ &= 0.49[17.23] + 0.49[22] + 0.02[21.87] \\ &= 22.1346 \end{aligned}$$

$$T_{10,1} = 0.49T_{9,0} + 0.49T_{11,0} + 0.02T_{10,0}$$

$$= 0.49(21.87) + 0.49(32.67) + 0.02(27)$$

$$= 27.2646$$

$$T_{11,1} = 0.49T_{10,0} + 0.49T_{12,0} + 0.02T_{11,0}$$

$$= 0.49(27) + 0.49(38.88) + 0.02(32.67)$$

$$= 32.9346$$

$$T_{12,1} = 0.49T_{11,0} + 0.49T_{13,0} + 0.02T_{12,0}$$

$$= 0.49(32.67) + 0.49(45.63) + 0.02(38.88)$$

$$= 39.1446$$

$$T_{13,1} = 0.49T_{12,0} + 0.49T_{14,0} + 0.02T_{13,0}$$

$$= 0.49(38.88) + 0.49(59.25) + 0.02(45.63)$$

$$= 45.8946$$

$$T_{14,1} = 0.49T_{13,0} + 0.49T_{15,0} + 0.02T_{14,0}$$

$$= 0.49(45.63) + 0.49(60.75) + 0.02(52.92)$$

$$= 53.1846$$

$$T_{15,1} = 0.49T_{14,0} + 0.49T_{16,0} + 0.02T_{15,0}$$

$$= 0.49(52.92) + 0.49(69.12) + 0.02(60.75)$$

$$= 61.0146$$

$$T_{16,1} = 0.49T_{15,0} + 0.49T_{17,0} + 0.02T_{16,0}$$

$$= 0.49(60.75) + 0.49(78.03) + 0.02(69.12)$$

$$= 69.3846$$

$$T_{17,1} = 0.49T_{16,0} + 0.49T_{18,0} + 0.02T_{17,0}$$

$$= 0.49(69.12) + 0.49(87.48) + 0.02(78.03)$$

$$= 78.2946$$

$$T_{18,1} = 0.49T_{17,0} + 0.49T_{19,0} + 0.02T_{18,0}$$

$$= 0.49(78.03) + 0.49(97.47) + 0.02(87.48)$$

$$= 87.7446$$

$$T_{19,1} = 0.49T_{18,0} + 0.49T_{20,0} + 0.02T_{19,0}$$

$$= 0.49(87.48) + 0.49(108) + 0.02(97.47)$$

$$= 97.7346$$

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Table For Solving up to $t = 0.02 \text{ hr}$ and $x = 6 \text{ cm}$

x	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0	3.3	3.6
$T(0)$	0	0.27	1.08	2.43	4.32	6.75	9.72	13.23	17.28	21.87	27.00	32.67	38.88
0.02	0	0.5846	1.3446	2.6946	4.5846	7.0146	9.9846	13.4946	17.5446	22.1346	27.2646	32.9346	39.1446

x	3.9	4.2	4.5	4.8	5.1	5.4	5.7	6.0
$T(0)$	45.63	52.92	60.75	69.12	78.03	87.48	97.47	108
0.02	45.8946	53.1846	61.0146	69.3846	78.2946	87.7446	97.7346	108