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16/ENG06/062
Mechanical Engineering
ENG382

Assignment 7

i The model for the temperature distribution in a rod of length L = 6cm is given as eqn

$$\frac{dT(x,t)}{dt} = C\frac{d^2T(x,t)}{dx^2}$$

where $C = 2.2cm^2/hr$

with the conditions and the temperature (°C)

$$T(x,0) = 3x^2$$

$$T(0,t) = 0$$

$$T(L,t) = -108$$

Using $\Delta t = 0.02hr$ and $\Delta x = 0.3cm$, obtain the temperature profile of the system for $0 \leq t \leq 0.3hr$

a Manually in tabular form Solving up to $t = 0.02hr$ and $x = 6cm$

Solution

Using explicit forward euler method

$$U_i^{k+1} = \gamma[U_{i+1}^k + U_{i-1}^k] + [1-2\gamma]U_i^k$$

when i = 1

$$U_{ij+1} = \gamma U_{2,j} + \gamma U_{0,j} + (1-2\gamma)U_{ij}$$

but $\gamma = \frac{C*dt}{(dx)^2} = \frac{2.2\times0.02}{0.3^2} = 0.49$

$$(1 - C2*0.49) = 0.02$$

For when i = 1 to 19

$$U_{1,j+1} = 0.49U_{0,j} + 0.49U_{2,j} + 0.02U_{1,j}$$

$$U_{2,j+1} = 0.49U_{1,j} + 0.49U_{3,j} + 0.02U_{2,j}$$

$$U_{3,j+1} = 0.49U_{2,j} + 0.49U_{4,j} + 0.02U_{3,j}$$

$u_{4,j+1} = 0.49u_{3,j} + 0.49u_{5,j} + 0.02u_{4,j}$

$u_{5,j+1} = 0.49u_{4,j} + 0.49u_{6,j} + 0.02u_{5,j}$

$u_{6,j+1} = 0.49u_{5,j} + 0.49u_{7,j} + 0.02u_{6,j}$

$u_{7,j+1} = 0.49u_{6,j} + 0.49u_{8,j} + 0.02u_{7,j}$

$u_{8,j+1} = 0.49u_{7,j} + 0.49u_{9,j} + 0.02u_{8,j}$

$u_{9,j+1} = 0.49u_{8,j} + 0.49u_{10,j} + 0.02u_{9,j}$

$u_{10,j+1} = 0.49u_{9,j} + 0.49u_{11,j} + 0.02u_{10,j}$

$u_{11,j+1} = 0.49u_{10,j} + 0.49u_{12,j} + 0.02u_{11,j}$

$u_{12,j+1} = 0.49u_{11,j} + 0.49u_{13,j} + 0.02u_{12,j}$

$u_{13,j+1} = 0.49u_{12,j} + 0.49u_{14,j} + 0.02u_{13,j}$

$u_{14,j+1} = 0.49u_{13,j} + 0.49u_{15,j} + 0.02u_{14,j}$

$u_{15,j+1} = 0.49u_{14,j} + 0.49u_{16,j} + 0.02u_{15,j}$

$u_{16,j+1} = 0.49u_{15,j} + 0.49u_{17,j} + 0.02u_{16,j}$

$u_{17,j+1} = 0.49u_{16,j} + 0.49u_{18,j} + 0.02u_{17,j}$

$u_{18,j+1} = 0.49u_{17,j} + 0.49u_{19,j} + 0.02u_{18,j}$

$u_{19,j+1} = 0.49u_{18,j} + 0.49u_{20,j} + 0.02u_{19,j}$

For the boundary condition

$T(x_1,0) = 3(0.3)^2 = 0.27$

$T(x_2,0) = 3(0.6)^2 = 1.08$

$T(x_3,0) = 3(0.9)^2 = 2.43$

$T(x_4,0) = 3(1.2)^2 = 4.32$

$T(x_5,0) = 3(1.5)^2 = 6.75$

$T(x_6,0) = 3(1.8)^2 = 9.72$

$T(x_7,0) = 3(2.1)^2 = 13.23$

$T(x_8,0) = 3(2.4)^2 = 17.28$

$T(x_9,0) = 3(2.7)^2 = 21.87$

$T(x_{10},0) = 3(3)^2 = 27$

$T(x_{11},0) = 3(3.3)^2 = 32.67$

$T(x_{12},0) = 3(3.6)^2 = 38.88$

$T(x_{13},0) = 3(3.9)^2 = 45.63$

$T(x_{14},0) = 3(4.2)^2 = 52.92$

$T(x_{15},0) = 3(4.5)^2 = 60.75$

$T(x_{16},0) = 3(4.8)^2 = 69.12$

$T(x_{17},0) = 3(5.1)^2 = 78.03$

$T(x_{18},0) = 3(5.4)^2 = 87.48$

$T(x_{19},0) = 3(5.7)^2 = 94.74$

$T(x_{20},0) = 3(6)^2 = 108$

$T(0,t) = 0 \quad T(L,t) = 108$

temperature has a range of 0 to 0.3hr with step size of 0.02hr. To get to 0.02hr, $j = 0$

when $j = 0$ [Replacing u with T]

$T_{1,1} = 0.49u_{0,0} + 0.49u_{2,0} + 0.02u_{1,0}$

$= 0.49(0) + 0.49(1.08) + 0.02(0.27)$

$= 0.5346$

$T_{2,1} = 0.49u_{1,0} + 0.49u_{3,0} + 0.02u_{2,0}$

$= 0.49(0.27) + 0.49(2.48) + 0.02(1.08)$

$= 1.3446$

$T_{3,1} = 0.49u_{2,0} + 0.49u_{4,0} + 0.02u_{3,0}$

$T_{3,1} = 0.49(1.08) + 0.49(4.32) + 0.02(2.43)$

$T_{3,1} = 4.5846$ 2.6946

$T_{4,1} = 0.49u_{3,0} + 0.49u_{5,0} + 0.02u_{4,0}$

$= 0.49(2.43) + 0.49(6.75) + 0.02(4.32)$

$= 4.5846$

$T_{5,1} = 0.49u_{4,0} + 0.49u_{6,0} + 0.02u_{5,0}$

$= 0.49(4.32) + 0.49(9.72) + 0.02(6.75)$

$= 7.0146$

$T_{6,1} = 0.49u_{5,0} + 0.49u_{7,0} + 0.02u_{6,0}$

$= 0.49[6.75) + 0.49[13.23) + 0.02(9.72)$

$T_{7,1} = 0.49T_{6,0} + 0.49T_{8,0} + 0.02T_{2,0}$

$= 0.49[9.727] + 0.49[17.25] + 0.02[13.2]$

$= 13.4946$

$T_{8,1} = 0.49T_{7,0} + 0.49T_{9,0} + 0.02T_{8,0}$

$= 0.49(13.28) + 0.49(21.87) + 0.02(17.28)$

$= 17.5446$

$T_{9,1} = 0.49T_{8,0} + 0.49T_{10,0} + 0.02T_{9,0}$

$= 0.49[17.28] + 0.49(22) + 0.02(21.87)$

$= 22.1346$

$T_{10,1} = 0.49T_{9,0} + 0.49T_{11,0} + 0.02T_{10,0}$

$= 0.49(21.87) + 0.49(32.67) + 0.02(27)$

$= 27.2646$

$T_{11,1} = 0.49T_{10,0} + 0.49T_{12,0} + 0.02T_{11,0}$

$= 0.49(27) + 0.49(38.88) + 0.02(32.67)$

$= 32.9346$

$T_{12,1} = 0.49T_{11,0} + 0.49T_{13,0} + 0.02T_{12,0}$

$= 0.49(32.67) + 0.49[45.68] + 0.02(38.88)$

$= 39.1446$

$T_{13,1} = 0.49T_{12,0} + 0.49T_{14,0} + 0.02T_{13,0}$

$= 0.49(38.88) + 0.49(39.2) + 0.02(45.68) = 45.8946$

$T_{14,1} = 0.49T_{13,0} + 0.49T_{15,0} + 0.02T_{14,0}$

$= 0.49(45.63) + 0.49(60.75) + 0.02(52.92)$

$= 53.1846$

$T_{15,1} = 0.49T_{14,0} + 0.49T_{16,0} + 0.02T_{15,0}$

$= 0.49(52.927) + 0.49(69.12) + 0.02(60.75)$

$= 61.0146$

$T_{16,1} = 0.49T_{15,0} + 0.49T_{17,0} + 0.02T_{16,0}$

$= 0.49(60.75) + 0.49(78.03) + 0.02(69.12)$

$= 69.3846$

$T_{17,1} = 0.49T_{16,0} + 0.49T_{18,0} + 0.02T_{17,0}$

$= 0.49[69.12] + 0.49(78.03) + 0.02(69.12)$

$= 69.3846$

$T_{18,1} = 0.49T_{17,0} + 0.49T_{19,0} + 0.02T_{18,0}$

$= 0.49(78.03) + 0.49[97.47] + 0.02[87.48]$

$= 87.7446$

$T_{19,1} = 0.49T_{18,0} + 0.49T_{20,0} + 0.02T_{19,0}$

$= 0.49(87.48) + 0.49(108) + 0.02(97.47)$

$= 97.7346$