

Question 21

(a) Mathematical modelling can be defined as the mathematical representation of a system and simulation of a system involves solving the model and obtaining its output variable for different values of its input variable or its input variable is changed from one value to another.

(b) By the use of Balance Law

(b) By differentiation

(c) By the use of Newton's law of cooling states

(c) From Newton's law of cooling states;

$$\frac{dT}{dt} \propto (T - T_A)$$

$$\frac{dT}{dt} = K(T - T_A)$$

$$\frac{dT}{(T - T_A)} = K dt$$

Integrating through

$$\ln(T - T_A) = Kt + C$$

$$(T - T_A) = e^{Kt+C}$$

$$T - T_A = e^{Kt} \cdot e^C$$

$$\text{But } e^C = T_0$$

$$T - T_A = T_0 e^{Kt} \quad \text{--- (1)}$$

Initially; at  $t=0$ ,  $T=10^\circ$  and actual temperature,  $T_A=25^\circ$

Equation one becomes:

$$10 - 25 = T_0 e^{K(0)}$$

$$-15 = T_0 \times 1 = T_0$$

$$T_0 = -15^\circ, \therefore T = T_A + T_0 e^{Kt} = 25 - 15e^{Kt} \quad \text{--- (2)}$$

at  $t = 5 \text{ minutes}$ ;  $T = 20^\circ\text{C}$

$$\text{from } T = 25 - 15e^{-kt}$$

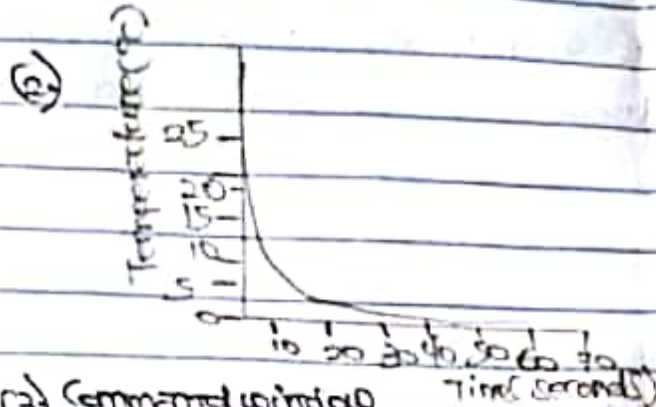
$$20 = 25 - 15e^{-kt}$$

$$-5 = -15e^{-kt}$$

$$\ln(1/3) = -kt$$

$$k = \ln(1/3)/5 = -0.22$$

$\therefore$  The model for the system becomes:  $T = 25 - 15e^{-0.22t}$



$$F(x) = 25 - 15 * \exp(-0.22 * A_2)$$

(3) Command window

```
clear  
clc  
close all  
t = 0:0.5:50  
T = 25 - 15 * exp(-0.22 * t)  
plot(t, T)  
xlabel('time(seconds)')  
ylabel('Temperature(°C)')
```

(4) The steady state temperature of the system =  $25^\circ\text{C}$

(5) At a temperature of  $25^\circ\text{C}$  is ~~not~~ observed of no change in temperature despite the increase in time, therefore the system is said to be stable at that temperature.

(6) For  $t = ?$  and  $T = 24.9^\circ\text{C}$

from the equation  $T = 25 - 15e^{-0.22t}$

$$24.9 = 25 - 15e^{-0.22t}$$

$$-0.1 = -15e^{-0.22t}$$

$$6.67 \times 10^{-3} = e^{-0.22t}$$

$$\ln 6.67 \times 10^{-3} / -0.22 = t$$

$$t = 22.76 \text{ mins}$$

0.76 minutes  $\rightarrow$  seconds

$$0.76 \times 60 = 45.6 \approx 46 \text{ seconds}$$

$\therefore$  The time required = 22 minutes and 46 seconds //