

NAME: EDUNBYE LEONARD MOTOSORETOLUNWA
MATIC NO: 171ENG02/061
DEPARTMENT: COMPUTER ENGINEERING

QUESTION 4

(a) Mathematical modelling is a mathematical representation of a system and the simulation of the system which involves solving the model and obtaining its output variable for different values of its input variable

(b) (i) By Differentiating e.g. Exponential Growth

(ii) By use of the Balance Law e.g. Missing problem

(iii) By use of Torricelli's Law e.g. Outflow of water through a hole

(c) Applying Newton's Law of Cooling

$$\frac{dT}{dt} = k(T - T_a)$$

dt

$$dT = k(T - T_a)dt$$

$$\frac{dT}{T - T_a} = kdt$$

$T - T_a$

$$\int \frac{dT}{T - T_a} = \int kdt$$

$$\ln |T - T_a| = kt + C^*$$

$$T - T_a = e^{kt} \cdot e^{C^*}$$

$$\text{Since } e^{C^*} = C$$

$$T - T_a = e^{kt} C$$

Initial temperature $T_0 = 10^\circ\text{C}$

Actual temperature of the system, $T_a = 25^\circ\text{C}$

$$10 = 25 + C e^{kt}$$

$$\text{At } T_0, t = 0$$

$$10 = 25 + C e^{k(0)}$$

$$10 = 25 + C$$

$$C = 10 - 25$$

$$C = -15$$

$$\therefore T(t) = T_1 - 15e^{kt}$$

At a temperature reading of 20°C after 5 minutes

$$20 = 25 - 15e^{k(5)}$$

$$-5 = -15e^{5k}$$

$$e^{5k} = \frac{-5}{-15}$$

$$= \frac{1}{3}$$

$$e^{5k} = 0.33$$

$$5k = \ln 0.33$$

$$5k = -1.1087$$

$$k = -0.22$$

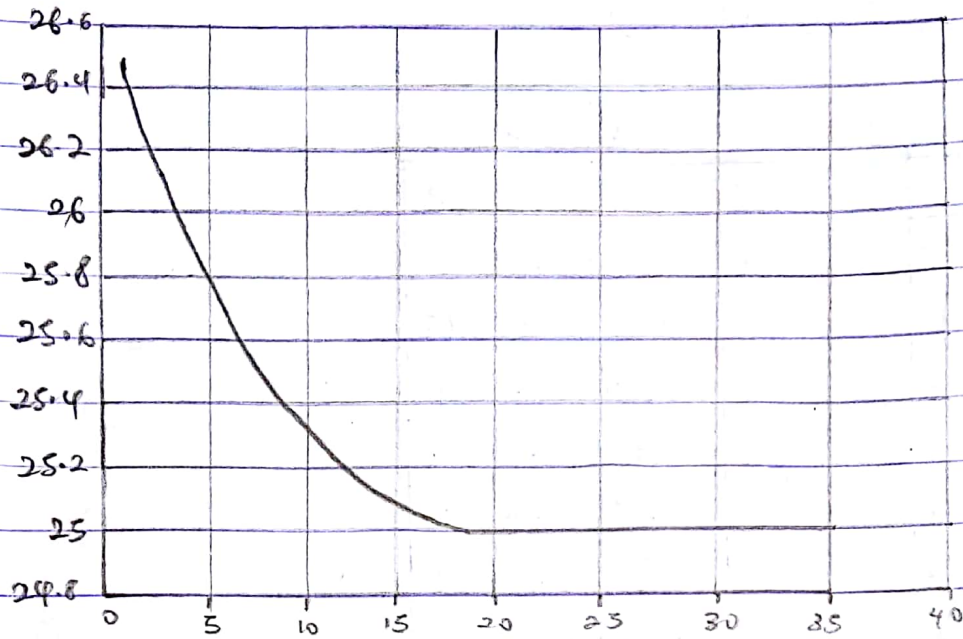
$$\therefore T_t = 25 - 15e^{-0.22t}$$

4(c)(ii) Output

$$f_x = 25 + 1.5 \cdot \text{DVP}(-0.22 \cdot A_2)$$

	A	B		A	B
1	0	26.5	28	39	25.00028
2	1.5	26.07839	29	40.5	25.0002
3	3	25.77528	30	42	25.00015
4	4.5	25.55737	31	43.5	25.0001
5	6	25.4009	32	45	25.00008
6	7.5	25.28807	33	46.5	25.00005
7	9	25.2071	34	48	25.00004
8	10.5	25.14889	35	49.5	25.00003
9	12	25.10704			
10	13.5	25.07695			
11	15	25.05532			
12	16.5	25.03977			
13	18	25.02859			
14	19.5	25.02056			
15	21	25.01478			
16	22.5	25.01063			
17	24	25.00764			
18	25.5	25.00549			
19	27	25.00395			
20	28.5	25.00284			
21	30	25.00204			
22	31.5	25.00147			
23	33	25.00105			
24	34.5	25.00076			
25	36	25.00055			
26	37.5	25.00039			

Graph



4(c)(ii)

OUTPUT

command window

Graph

clear

clc

close all

syms t T

t = 0:0.5:50

$T = 25 - 1.5 * \exp(-0.22 * t)$

T2 = subs(T)

plot(t, T2)

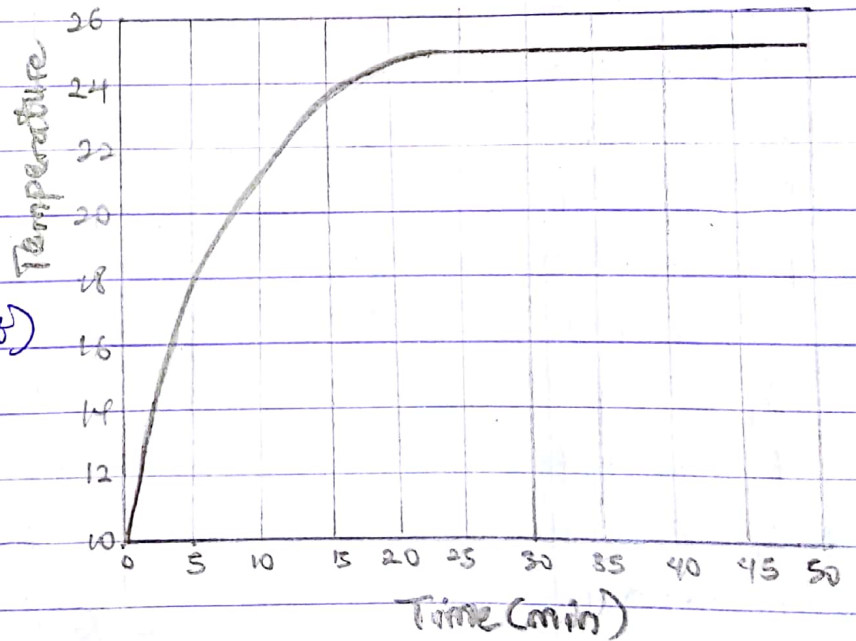
grid on

grid minor

axis tight

xlabel('Time (min)')

ylabel('Temperature')



(iv) The steady-state temperature of the system is 25°C

(v) The system is stable when the temperature is at 25°C .

(vi) The time required for the thermometer to reach 24.9°C is 23 minutes