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Mechanical Engg.

17/ENG061069

Eng 282

1. A mathematical model is a mathematical representation of a system in order to study its behaviour

b. Methods of obtaining a model

• Differential equation

• Use of balance law

c. From Newton law of cooling.

$$\frac{dT}{dt} = k(T - T_m)$$

$$dT = k(T - T_m)dt$$

$$\frac{dT}{T - T_m} = kdt$$

(Integrating both sides)

$$\ln(T - T_m) = kt + C$$

$$T - T_m = e^{kt+C}$$

$$T - T_m = e^{kt} \cdot e^C$$

$$\text{Let } e^C = C$$

$$T - T_m = e^{kt} \cdot C$$

$$T - T_m = C e^{kt}$$

$$T = C e^{kt} + T_m$$

$$\text{At } t = 0, T = 10^\circ\text{C}, T_m = 25^\circ\text{C}$$

$$10 = C e^{k \cdot 0} + 25$$

$$10 = C + 25$$

$$C = 10 - 25$$

$$C = -15$$

$$T = -15 e^{kt} + 25$$

$$\text{At } t = 5, T = 20^\circ\text{C}$$

$$20 = -15 e^{k(5)} + 25$$

$$20 - 25 = -15 e^{k(5)}$$

$$\frac{15}{-15} = \frac{-15}{-15} e^{k(5)}$$

$$\Rightarrow \frac{1}{3} = e^{5k}$$

$$5k = \ln(0.333)$$

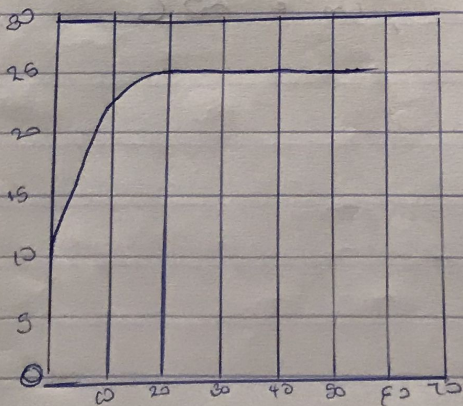
$$\frac{5k}{5} = \frac{\ln(0.333)}{5}$$

$$k = -0.22$$

$$\therefore T_0 = 25 - 15e^{-0.22t}$$

ii Using Microsoft excel

- type in 't' for column 1
- type in 'T' for column 2
- ~~for~~ column 1, type '0' under t
- On home, select fill under editing
- select series, choose columns, insert a step value of 1 and a step value of 60 in enter
- then in column 2 under T, type = 25 - 15e<sup>2p(-0.22)</sup> then enter
- double click the dot at the bottom right of the cell.
- highlight columns 1 and 2.
- Go to insert and pick a graph type.



Using Matlab.

• Command.

• Clear

clc

clear all

t = (0:1:60)

T = 25 - 15 \* exp(-0.22 \* t)

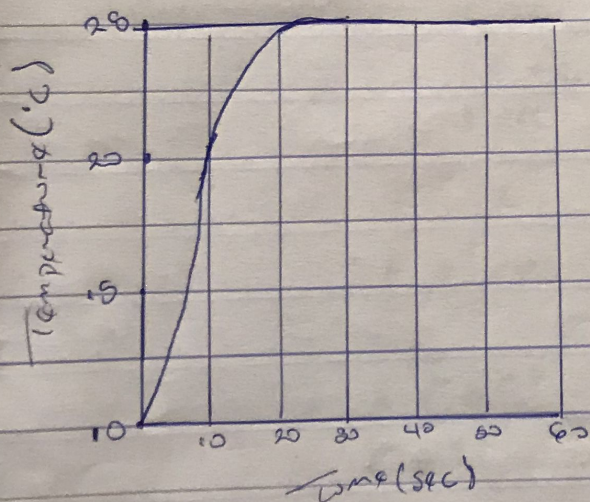
Plot (t, T)

grid on.

grid minor

label ('Temp (°C)')

label ('Temperature (°C)')



• Letting the dynamic responses, the steady state is 25°C at 25 minutes.

• Using the developed model equation, the temperature at the thermometer at  $t \rightarrow 0 = 10^\circ\text{C}$ .