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16/ENG06/1061
MECHANICAL 300L

ASSIGN 7

The model for the temperature distribution in a rod of length L cm is as given in eqn (1)

$$\frac{\partial T(x,t)}{\partial t} = C\frac{\partial^2 T(x,t)}{\partial x^2}$$

Where $C = \frac{2.2\,cm^2}{hr}$

With the Conditions that the temp (°C):

$T(x,0) = 3x^2$

$T(0,t) = 0$

$T(L,t) = 10$

Using $\Delta t = 0.02hr$ and $\Delta x = 0.3cm$

Obtain the temp profile of the System for $0 \le t \le 0.3hr$

a) Manually in tabular form Showing Up to $t = 0.02hr$ and $x = 6cm$

b) With the aid of Microsoft Excel, in tabular 3D graphical forms and

c) With the aid of MATLAB in tabular 3D graphical forms.

Solution

(a) Using Explicit forward Euler method.

$$u_i^{k+1} = r u_{i-1}^k + (1-2r) u_i^k + r u_{i+1}^k$$

$$r = \frac{c(\Delta t)}{(\Delta x)^2} = \frac{2.2 \times 0.02}{0.3^2}$$

$$r = 0.49$$

$$u_i^{k+1} = 0.49 u_{i-1,k} + 0.02 u_{i,k} + 0.49 u_{i+1,k}$$

For $k = 1$ to 19

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1→	$u_1^{k+1} = 0.49 u_{0,k} + 0.02 u_{1,k} + 0.49 u_{2,k}$
2→	$u_2^{k+1} = 0.49 u_{1,k} + 0.02 u_{2,k} + 0.49 u_{3,k}$
3→	$u_3^{k+1} = 0.49 u_{2,k} + 0.02 u_{3,k} + 0.49 u_{4,k}$
4→	$u_4^{k+1} = 0.49 u_{3,k} + 0.02 u_{4,k} + 0.49 u_{5,k}$
5→	$u_5^{k+1} = 0.49 u_{4,k} + 0.02 u_{5,k} + 0.49 u_{6,k}$
6→	$u_6^{k+1} = 0.49 u_{5,k} + 0.02 u_{6,k} + 0.49 u_{7,k}$
7→	$u_7^{k+1} = 0.49 u_{6,k} + 0.02 u_{7,k} + 0.49 u_{8,k}$
8→	$u_8^{k+1} = 0.49 u_{7,k} + 0.02 u_{8,k} + 0.49 u_{9,k}$
9→	$u_9^{k+1} = 0.49 u_{8,k} + 0.02 u_{9,k} + 0.49 u_{10,k}$
10→	$u_{10}^{k+1} = 0.49 u_{9,k} + 0.02 u_{10,k} + 0.49 u_{11,k}$
11→	$u_{11}^{k+1} = 0.49 u_{10,k} + 0.02 u_{11,k} + 0.49 u_{12,k}$
12→	$u_{12}^{k+1} = 0.49 u_{11,k} + 0.02 u_{12,k} + 0.49 u_{13,k}$
13→	$u_{13}^{k+1} = 0.49 u_{12,k} + 0.02 u_{13,k} + 0.49 u_{14,k}$
14→	$u_{14}^{k+1} = 0.49 u_{13,k} + 0.02 u_{14,k} + 0.49 u_{15,k}$
15→	$u_{15}^{k+1} = 0.49 u_{14,k} + 0.02 u_{15,k} + 0.49 u_{16,k}$

$$U_{16}^{K+1} = 0.49\,U_{15,K} + 0.02\,U_{16,K} + 0.49\,U_{17,K}$$

$$U_{17}^{K+1} = 0.49\,U_{16,K} + 0.02\,U_{17,K} + 0.49\,U_{18,K}$$

$$U_{18}^{K+1} = 0.49\,U_{17,K} + 0.02\,U_{18,K} + 0.49\,U_{19,K}$$

$$U_{19}^{K+1} = 0.49\,U_{18,K} + 0.02\,U_{19,K} + 0.49\,U_{20,K}$$

Boundary Condition

$T(x,0) = 3x^2$; $0 < (0.3)6$ cm

$T(x_1,0) = 3(0.3)^2 = 0.27$	$T(x_{16},0) = 3(4.8)^2 = 69.12$
$T(x_2,0) = 3(0.6)^2 = 1.08$	$T(x_{17},0) = 3(5.1)^2 = 78.03$
$T(x_3,0) = 3(0.9)^2 = 2.43$	$T(x_{18},0) = 3(5.4)^2 = 87.48$
$T(x_4,0) = 3(1.2)^2 = 4.32$	$T(x_{19},0) = 3(5.7)^2 = 94.74$
$T(x_5,0) = 3(1.5)^2 = 6.75$	$T(x_{20},0) = 3(6)^2 = 108$
$T(x_6,0) = 3(1.8)^2 = 9.72$	
$T(x_7,0) = 3(2.4)^2 = 13.23$	$T(0,t) = 0$,
$T(x_8,0) = 3(2.4)^2 = 17.28$	$T(L,t) = 108$
$T(x_9,0) = 3(2.7)^2 = 21.87$	
$T(x_{10},0) = 3(3)^2 = 27$	Temperature has a
$T(x_{11},0) = 3(3.3)^2 = 32.67$	range of 0 to 0.3 h
$T(x_{12},0) = 3(3.6)^2 = 38.88$	with step size of
$T(x_{13},0) = 3(3.9)^2 = 45.63$	0.02 hr.
$T(x_{14},0) = 3(4.2)^2 = 52.92$	
$T(x_{15},0) = 3(4.5)^2 = 60.75$	

For 0.02hr , K=0

$T_{1,1} = 0.49U_{0,0} + 0.49_{2,0} + 0.02U_{1,0}$

$= 0.49(0) + 0.49(1.08) + 0.02(0.27) = 0.5346$

$T_{2,1} = 0.49U_{1,0} + 0.49U_{3,0} + 0.02U_{2,0}$

$= 0.49(0.27) + 0.49(2.43) + 0.02(1.08) = 1.3446$

$T_{3,1} = 0.49U_{2,0} + 0.49U_{4,0} + 0.02U_{3,0}$

$= 0.49(1.08) + 0.49(4.32) + 0.02(2.43) = 2.6946$

$T_{4,1} = 0.49U_{3,0} + 0.49U_{5,0} + 0.02U_{4,0}$

$= 0.49(2.43) + 0.49(6.75) + 0.02(4.32) = 4.5846$

$T_{5,1} = 0.49U_{4,0} + 0.49_{6,0} + 0.02U_{5,0}$

$= 0.49(4.32) + 0.49(9.72) + 0.02(6.75) = 7.0146$

$T_{6,1} = 0.49U_{5,0} + 0.49U_{7,0} + 0.02U_{6,0}$

$= 0.49(6.75) + 0.49(13.23) + 0.02(9.72) = 9.9846$

$T_{7,1} = 0.49U_{6,0} + 0.49(18,0 + 0.02U_{7,0}$

$= 0.49(9.72) + 0.49(17.25) + 0.02(13.23) = 13.4946$

$T_{8,1} = 0.49U_{7,0} + 0.49U_{9,0} + 0.02U_{8,0}$

$= 0.49(13.28) + 0.49(21.87) + 0.02(17.28] = 17.5446$

$T_{9,1} = 0.49U_{8,0} + 0.49U_{10,0} + 0.02U_{9,0}$

$= 0.49(17.28) + 0.49(27) + 0.02(21.87) = 22.1346$

$T_{10,1} = 0.49U_{9,0} + 0.49U_{11,0} + 0.02U_{10,0}$

$= 0.4(21.87] + 0.49(32.67) + 0.02(27)$

$= 27.2646$

$T_{11,1} = 0.49\,U_{10,0} + 0.49\,U_{12,0} + 0.02\,U_{11,0}$

$= 0.49(27) + 0.49(38.88) + 0.02(32.67) = 32.9346$

$T_{12,1} = 0.49\,U_{11,0} + 0.49\,U_{13,0} + 0.02\,U_{12,0}$

$= 0.49(32.67) + 0.49(45.63) + 0.02(38.88) = 39.1446$

$T_{13,1} = 0.49\,U_{12,0} + 0.49\,U_{14,0} + 0.02\,U_{13,0}$

$= 0.49(38.88) + 0.49(59.2) + 0.02(45.63) = 45.8946$

$T_{14,1} = 0.49\,U_{13,0} + 0.49\,U_{15,0} + 0.02\,U_{14,0}$

$= 0.49(45.63) + 0.49(60.75) + 0.02(52.92) = 53.1846$

$T_{15,1} = 0.49(U_{14,0} + 0.49\,U_{16,0} + 0.02\,U_{15,0}$

$= 0.49(52.92) + 0.49(69.12) + 0.02(60.75) = 61.0146$

$T_{16,1} = 0.49\,U_{15,0} + 0.49\,U_{17,0} + 0.02\,U_{16,0}$

$= 0.49(60.75) + 0.49(78.03) + 0.02(69.12) = 69.3846$

$T_{17,1} = 0.49\,U_{16,0} + 0.49\,U_{18,0} + 0.02\,U_{17,0}$

$= 0.49(69.12) + 0.49(87.48) + 0.02(78.03) = 78.2946$

$T_{18,1} = 0.49\,U_{17,0}(78.03) + 0.49\,U_{19,0} + 0.02\,U_{18,0}$

$= 0.49(78.03) + 0.49(97.49) + 0.02(87.48) = 87.7446$

$T_{19,1} = 0.49\,U_{18,0}(87.48) + 0.49(U_{20,0} + 0.02\,U_{19,0}$

$= 0.49(87.48) + 0.49(108) + 0.02(97.49)$

$= 97.7346$