~~Q1~~

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Assignment V I

a) Define Mathematical Modelling

Mathematical modelling is defined as the act of describing a system and its process using mathematical concept and language. It is the process of setting up a model, solving its mathematically and interpreting the result in physical or other terms.

b) Outline two methods of obtaining mathematical model for engineering system.

i. Solution by a mathematical method.

ii Transition from the physical solution (physical system) to its methematical formulation (its mathematical model)

iii interpreting the result and simulating the model

c) A thermometer that initially reads $10°C$ is used to measure the temperature of of a system. The temperature of the thermometer is discovered to be $20°C$ after 5mins of inserting it into the system. If the actual temperature of the system is $25°C$. (i) develop a model for the system.

ii Simulate the developed model for time $t=0$ to $t=60$mins using a step of 1min with the aid of MS Excel.

iii Obtain the dynamic response of the system with the aid of MATLAB without using syms command, for $t=0$ to 60 using a system step of 1min

iv Using either the dynamic response, write the steady state temperature of the system and

v Using the developed model equation, evaluate the temperature of the thermometer at $t \to \infty$

Solution:

T_A(Actual temperature) $= 25°C$

T (initial temperature) $= 10°C$

Using Newton's Law of Cooling,

$$\frac{dT}{dt} = K(T - T_A)$$

Recall; the equation is variable separable

$$\frac{dT}{(T-T_A)} = Kdt$$

Integrating both sides;

$\ln(T - T_A) = kt + c$

$T - T_A = e^{(kt+c)}$

$T - T_A = e^{kt} \cdot e^{c} \Rightarrow e^{c} = c$

$T - T_A = e^{kt} \cdot c$

$T - T_A = ce^{kt}$

$T = ce^{kt} + T_A$

at $T_A = 25°C$, $T = 10°C$, $t = 0$ mins

$10 = ce^{k.0} + 25$

$10 = ce^{0} + 25 \Rightarrow e^{0} = 1$

$10 = c + 25$

$c = 10 - 25$

$c = -15$

$\underline{T = T_A - 15e^{kt}}$

at $T = 20°C$; $T_A = 25°C$; $t = 5$ mins

$T = T_A - 10e^{kt}$

$20 = 25 - 10e^{k.5}$

$20 - 25 = -15e^{5k}$

$-5 = -15e^{5k}$

$\frac{-5}{-15} = e^{5k}$

$\frac{1}{3} = e^{5k}$

$\ln 1/3 = 5k$

$-1.099 = 5k$

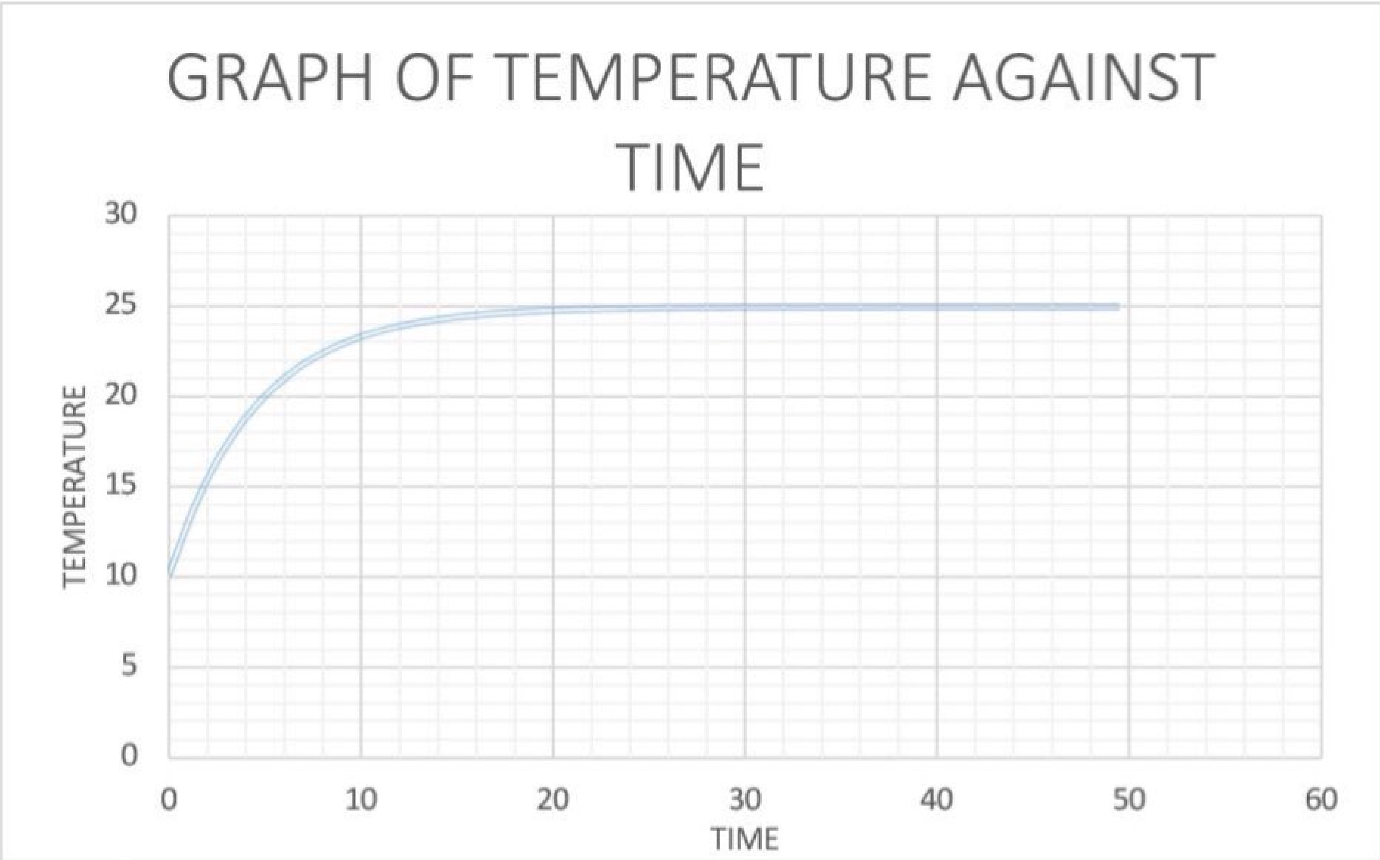
$k = \frac{-1.099}{5}$

$k = -0.22$

$\Rightarrow \underline{T = 25 - 15e^{-0.22t}}$

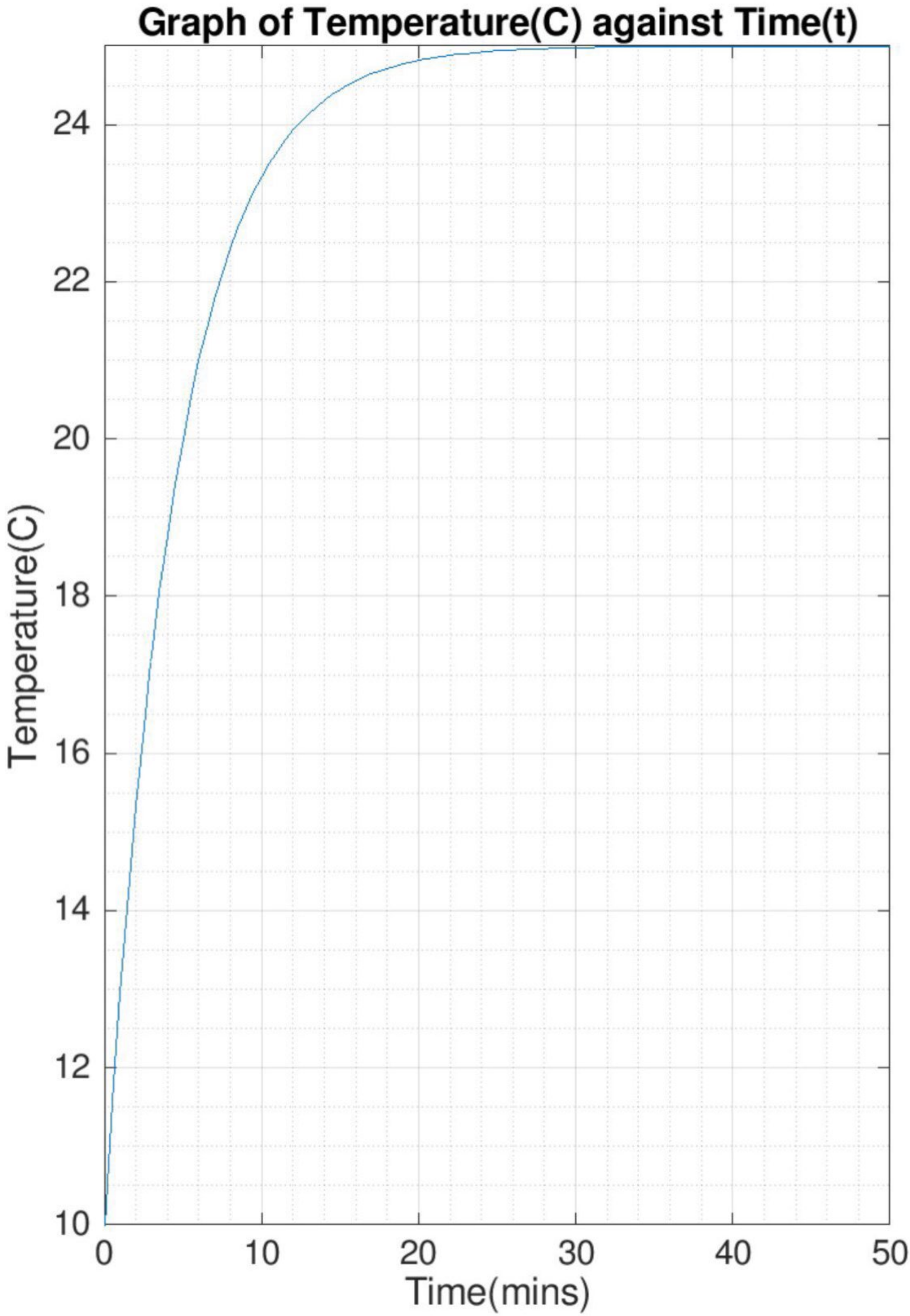
ii

X(TIME)	Y=25-15*EXP(-0.22*A2)
0	10
1.5	14.216144
3	17.24722998
4.5	19.42634963
6	20.99297047
7.5	22.11925137
9	22.92896144
10.5	23.51108123
12	23.92958096
13.5	24.23045035
15	24.44675249
16.5	24.60225723
18	24.71405329
19.5	24.79442612
21	24.85220806
22.5	24.89374887
24	24.92361354
25.5	24.94508396
27	24.96051956
28.5	24.97161657
30	24.97959448
31.5	24.98532999
33	24.98945338
34.5	24.99241778
36	24.99454897
37.5	24.99608112
39	24.99718263
40.5	24.99797452
42	24.99854384
43.5	24.99895313
45	24.99924738
46.5	24.99945892
48	24.99961101
49.5	24.99972034


GRAPH OF TEMPERATURE AGAINST TIME
TEMPERATURE
TIME
30
25
20
15
10
5
0
0
10
20
30
40
50
60

iii commandwindow

```
clearall
clc
close all
~~t = 0:1:60~~ t = 0:0.5:50
T= 25-15*exp(-0.22*t)
Tn = subs(T)
plot(t, Tn)
xlabel('Time(mins)')
ylabel ('Temperature(C)')
title('Graph of Temperature(T) against Time (t)')
grid on
grid minor
axis tight
```


Graph of Temperature(C) against Time(t)
Temperature(C)
Time(mins)
10
12
14
16
18
20
22
24
0
10
20
30
40
50

iv The study state value of the temperature is 25°C at 30 mins of the exponential approach.

v As t tends to infinity, The temperature approaches the steady state value which is 25°C.

vi $T = 24.9$

Recall

$$T = T_A - Ce^{kt}$$

$$24.9 = 25 - 15e^{-0.22t}$$

$$24.9 - 25 = -15e^{-0.22t}$$

$$-0.1 = -15e^{-0.22t}$$

$$\frac{-0.1}{-15} = e^{-0.22t}$$

$$\frac{1}{150} = e^{-0.22t}$$

$$\ln 1/150 = -0.22t$$

$$t = \frac{-5.01}{-0.22} = \underline{22.77\,min}$$