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16/ENG 06/043

Mechanical

ENG 382

(1) The model for the temperature distribution in a rod of length  $l = 6\text{cm}$  is as given in eqn 1;

$$\frac{dT}{dt}(x, t) = \frac{d^2 T}{dx^2}(x, t)$$

where  $C = 2.2 \text{ cm}^2/\text{hr}$

with the conditions and the temperature ( $^{\circ}\text{C}$ )

$$T(x, 0) = 3x^2$$

$$T(0, t) = 0$$

$$T(l, t) = -108$$

using  $\Delta T = 0.02\text{hr}$  and  $\Delta x = 0.3\text{cm}$ , obtain the temperature profile of the system for  $0 \leq t \leq 0.3\text{hr}$

a) Manually, in tabular form solving up to  $t = 0.02\text{hr}$  and  $x = 6\text{cm}$

Solution

using explicit forward Euler method

when  $T = 1$

$$U_{i,j+1} = \gamma U_{i-1,j} + \gamma U_{i+1,j} + (1-2\gamma)U_{i,j}$$

$$\text{but } \gamma = \frac{C \cdot \Delta t}{(\Delta x)^2} = \frac{2.2 \times 0.02}{0.3^2} = 0.49$$

$$[1 - (2 \times 0.49)] = 0.02$$

Rewriting the explicit forward Euler method,

For when  $i = 1$  to  $19$

$$U_{1,j+1} = 0.49U_{0,j} + 0.49U_{2,j} + 0.02U_{1,j}$$

$$U_{2,j+1} = 0.49U_{1,j} + 0.49U_{3,j} + 0.02U_{2,j}$$

$$U_{3,j+1} = 0.49U_{2,j} + 0.49U_{4,j} + 0.02U_{3,j}$$

$$U_{4,j+1} = 0.49U_{3,j} + 0.49U_{5,j} + 0.02U_{4,j}$$

$$U_{5,j+1} = 0.49U_{4,j} + 0.49U_{6,j} + 0.02U_{5,j}$$

$$U_{6,j+1} = 0.49U_{5,j} + 0.49U_{7,j} + 0.02U_{6,j}$$

$$U_{7,j+1} = 0.49U_{6,j} + 0.49U_{8,j} + 0.02U_{7,j}$$



$$U_{8,j+1} = 0.49U_{7,j} + 0.49U_{9,j} + 0.02U_{8,j}$$

$$U_{9,j+1} = 0.49U_{8,j} + 0.49U_{10,j} + 0.02U_{9,j}$$

$$U_{10,j+1} = 0.49U_{9,j} + 0.49U_{11,j} + 0.02U_{10,j}$$

$$U_{11,j+1} = 0.49U_{10,j} + 0.49U_{12,j} + 0.02U_{11,j}$$

$$U_{12,j+1} = 0.49U_{11,j} + 0.49U_{13,j} + 0.02U_{12,j}$$

$$U_{13,j+1} = 0.49U_{12,j} + 0.49U_{14,j} + 0.02U_{13,j}$$

$$U_{14,j+1} = 0.49U_{13,j} + 0.49U_{15,j} + 0.02U_{14,j}$$

$$U_{15,j} = 0.49U_{14,j} + 0.49U_{16,j} + 0.02U_{15,j}$$

$$U_{16,j} = 0.49U_{15,j} + 0.49U_{17,j} + 0.02U_{16,j}$$

$$U_{17,j} = 0.49U_{16,j} + 0.49U_{18,j} + 0.02U_{17,j}$$

$$U_{18,j} = 0.49U_{17,j} + 0.49U_{19,j} + 0.02U_{18,j}$$

$$U_{19,j} = 0.49U_{18,j} + 0.49U_{20,j} + 0.02U_{19,j}$$

For the boundary condition

$T(x,0) = 3x^2$  with  $x$  ranging from 0 to 6cm with step size of 0.3

$$T(x,0) = 3x^2 = 3(0.3)^2 = 0.27$$

$T(x,$

$$T(x_0,0) = 3x^2 = 3(0.3)^2 = 0.27$$

$$T(x_1,0) = 3x^2 = 3(0.6)^2 = 1.08$$

$$T(x_2,0) = 3x^2 = 3(1.2)^2 = 4.32$$

$$T(x_3,0) = 3x^2 = 3(1.5)^2 = 6.75$$

$$T(x_4,0) = 3x^2 = 3(1.8)^2 = 9.72$$

$$T(x_5,0) = 3x^2 = 3(2.1)^2 = 13.23$$

$$T(x_6,0) = 3x^2 = 3(2.4)^2 = 17.28$$

$$T(x_7,0) = 3x^2 = 3(2.7)^2 = 21.87$$

$$T(x_8,0) = 3x^2 = 3(3.0)^2 = 27$$

$$T(x_9,0) = 3x^2 = 3(3.3)^2 = 32.67$$

$$T(x_{10},0) = 3x^2 = 3(3.6)^2 = 38.88$$

$$T(x_{11},0) = 3x^2 = 3(3.9)^2 = 45.63$$

$$T(x_{12},0) = 3x^2 = 3(4.2)^2 = 52.92$$

$$T(x_{13},0) = 3x^2 = 3(4.5)^2 = 60.75$$

$$T(x_{14},0) = 3x^2 = 3(4.8)^2 = 69.12$$

$$T(x_{15},0) = 3x^2 = 3(5.1)^2 = 78.03$$

$$T(x_{16},0) = 3x^2 = 3(5.4)^2 = 87.48$$

$$T(x_{17},0) = 3x^2 = 3(5.7)^2 = 97.47$$

$$T(x_{18},0) = 3x^2 = 3(6.0)^2 = 108$$

$$T(0,t) = 0, T(6,t) = 108$$

temperature has a range of 0 to 0.3hr with step size of 0.02hr to get to 0.02hr,  $j=0$

when  $j=0$  (replacing  $U$  with  $T$ )

$$T_{1,1} = 0.49U_{0,0} + 0.49U_{2,0} + 0.02U_{1,0}$$

$$= 0.49(0) + 0.49(1.08) + 0.02(0.27)$$



$$= 0.5346$$

$$\begin{aligned}\bar{T}_{2,1} &= 0.49u_{1,0} + 0.49u_{3,0} + 0.02u_{2,0} \\ &= 0.49(0.27) + 0.49(2.43) + 0.02(1.08) \\ &= 1.3446\end{aligned}$$

$$\begin{aligned}\bar{T}_{3,1} &= 0.49u_{2,0} + 0.49u_{4,0} + 0.02u_{3,0} \\ &= 0.49(1.08) + 0.49(4.32) + 0.02(2.43) \\ &= 2.6976\end{aligned}$$

$$\begin{aligned}\bar{T}_{4,1} &= 0.49u_{3,0} + 0.49u_{5,0} + 0.02u_{4,0} \\ &= 0.49(2.43) + 0.49(6.75) + 0.02(4.32) \\ &= 4.5846\end{aligned}$$

$$\begin{aligned}\bar{T}_{5,1} &= 0.49u_{4,0} + 0.49u_{6,0} + 0.02u_{5,0} \\ &= 0.49(7.32) + 0.49(9.72) + 0.02(0.75) \\ &= 7.0146\end{aligned}$$

$$\begin{aligned}\bar{T}_{6,1} &= 0.49\bar{T}_{5,0} + 0.49\bar{T}_{7,0} + 0.02\bar{T}_{6,0} \\ &= 0.49(6.75) + 0.49(13.23) + 0.02(9.72) \\ &= 9.9846\end{aligned}$$

$$\begin{aligned}\bar{T}_{7,1} &= 0.49\bar{T}_{6,0} + 0.49\bar{T}_{8,0} + 0.02\bar{T}_{7,0} \\ &= 0.49(9.72) + 0.49(17.27) + 0.02(13.23) \\ &= 13.4946\end{aligned}$$

$$\begin{aligned}\bar{T}_{8,1} &= 0.49\bar{T}_{7,0} + 0.49\bar{T}_{9,0} + 0.02\bar{T}_{8,0} \\ &= 0.49(13.23) + 0.49(21.87) + 0.02(17.28) \\ &= 17.5446\end{aligned}$$

$$\begin{aligned}\bar{T}_{9,1} &= 0.49\bar{T}_{8,0} + 0.49\bar{T}_{10,0} + 0.02\bar{T}_{9,0} \\ &= 0.49(17.28) + 0.49(22) + 0.02(21.87) \\ &= 22.1346\end{aligned}$$

$$\begin{aligned}\bar{T}_{10,1} &= 0.49\bar{T}_{9,0} + 0.49\bar{T}_{11,0} + 0.02\bar{T}_{10,0} \\ &= 0.49(21.87) + 0.49(32.67) + 0.02(22) \\ &= 27.2646\end{aligned}$$

$$\begin{aligned}\bar{T}_{11,1} &= 0.49\bar{T}_{10,0} + 0.49\bar{T}_{12,0} + 0.02\bar{T}_{11,0} \\ &= 0.49(29) + 0.49(38.88) + 0.02(32.67) \\ &= 32.9346\end{aligned}$$

$$\begin{aligned}\bar{T}_{12,1} &= 0.49\bar{T}_{11,0} + 0.49\bar{T}_{13,0} + 0.02\bar{T}_{12,0} \\ &= 0.49(32.67) + 0.49(45.63) + 0.02(38.88) \\ &= 39.1446\end{aligned}$$



$$\begin{aligned} T_{13,1} &= 0.49 T_{12,0} + 0.49 T_{14,0} + 0.02 T_{13,0} \\ &= 0.49 (38.88) + 0.49 (59.2) + 0.02 (45.63) \\ &= 45.8946 \end{aligned}$$

$$\begin{aligned} T_{14,1} &= 0.49 T_{13,0} + 0.49 T_{15,0} + 0.02 T_{14,0} \\ &= 0.49 (45.63) + 0.49 (60.75) + 0.02 (52.92) = 53.1846 \end{aligned}$$

$$\begin{aligned} T_{15,1} &= 0.49 T_{14,0} + 0.49 T_{16,0} + 0.02 T_{15,0} \\ &= 0.49 (52.92) + 0.49 (69.12) + 0.02 (60.75) = 61.0146 \end{aligned}$$

$$\begin{aligned} T_{16,1} &= 0.49 T_{15,0} + 0.49 T_{17,0} + 0.02 T_{16,0} \\ &= 0.49 (60.75) + 0.49 (78.03) + 0.02 (69.12) = 69.3846 \end{aligned}$$

$$\begin{aligned} T_{17,1} &= 0.49 T_{16,0} + 0.49 T_{18,0} + 0.02 T_{17,0} \\ &= 0.49 (69.12) + 0.49 (87.48) + 0.02 (78.03) \\ &= 78.2946 \end{aligned}$$

$$\begin{aligned} T_{18,1} &= 0.49 T_{17,0} + 0.49 T_{19,0} + 0.02 T_{18,0} \\ &= 0.49 (78.03) + 0.49 (97.47) + 0.02 (87.48) \\ &= 87.7446 \end{aligned}$$

$$\begin{aligned} T_{19,1} &= 0.49 T_{18,0} + 0.49 T_{20,0} + 0.02 T_{19,0} \\ &= 0.49 (87.48) + 0.49 (108) + 0.02 (97.47) \\ &= 97.7346 \end{aligned}$$

Table for solving up to  $t = 0.02$  hr and  $x = 6$  cm

$x$	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0
$T(0)$	0	0.27	1.08	2.43	4.32	6.75	9.72	13.23	17.28	21.81	26.00
0.02	0	0.5346	1.3446	2.6946	4.5846	7.0146	9.9846	13.4946	17.5446	22.1346	27.2646

3.3	3.6	3.9	4.2	4.5	4.8	5.1	5.4	5.7	6.0
32.67	38.88	45.63	52.92	60.75	69.12	78.03	87.48	97.47	108
32.9346	39.1446	45.8946	53.1846	61.0146	69.3846	78.2946	87.7446	97.7346	108