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ELECT/ELECT
ENG 382

(1) The model for the temperature distribution in a rod of length $l = 6cm$ is as given in eqn 1;

$$\frac{dT(x,t)}{dt} = \frac{d^2T(x,t)}{dx^2}$$

where; $C = 2.2\ cm^2/hr$

with the conditions and the temperature ($^\circ C$)

$$T(x,0) = 3x^2$$
$$T(0,t) = 0$$
$$T(l,t) = -108$$

using $\Delta T = 0.02hr$ and $\Delta x = 0.3\ cm$, obtain the temperature profile of the system for $0 \leq t \leq 0.3hr$

(2) Manually, in tabular form solving up to $t = 0.02hr$ and $x = 6cm$

Solution

using explicit forward euler method

$$u_T^{k+1} = \gamma\left[u_{T+1}^{k} + u_{T-1}^{k}\right] + \left[1-2\gamma\right]u_T^k$$

when $T = 1$

$$u_{1,j+1} = \gamma u_{2,j} + \gamma u_{0,j} + (1-2\gamma)u_{1,j}$$

but $\gamma = \frac{C \cdot dt}{(dx)^2} = \frac{2.2 \times 0.02}{0.3^2} = 0.49$

$$\left[1 - (2 \times 0.49)\right] = 0.02$$

Rewriting the explicit forward euler method,

For when $T = 1$ to 19

$$u_{1,j+1} = 0.49u_{0,j} + 0.49u_{2,j} + 0.02u_{1,j}$$
$$u_{2,j+1} = 0.49u_{1,j} + 0.49u_{3,j} + 0.02u_{2,j}$$
$$u_{3,j+1} = 0.49u_{2,j} + 0.49u_{4,j} + 0.02u_{3,j}$$
$$u_{4,j+1} = 0.49u_{3,j} + 0.49u_{5,j} + 0.02u_{4,j}$$
$$u_{5,j+1} = 0.49u_{4,j} + 0.49u_{6,j} + 0.02u_{5,j}$$
$$u_{6,j+1} = 0.49u_{5,j} + 0.49u_{7,j} + 0.02u_{6,j}$$
$$u_{7,j+1} = 0.49u_{6,j} + 0.49u_{8,j} + 0.02u_{7,j}$$

$U_{8,j+1} = 0.49U_{7,j} + 0.49U_{9,j} + 0.02U_{8,j}$

$U_{9,j+1} = 0.49U_{8,j} + 0.49U_{10,j} + 0.02U_{9,j}$

$U_{10,j+1} = 0.49U_{9,j} + 0.49U_{11,j} + 0.02U_{10,j}$

$U_{11,j+1} = 0.49U_{10,j} + 0.49U_{12,j} + 0.02U_{11,j}$

$U_{12,j+1} = 0.49U_{11,j} + 0.49U_{13,j} + 0.02U_{12,j}$

$U_{13,j+1} = 0.49U_{12,j} + 0.49U_{14,j} + 0.02U_{13,j}$

$U_{14,j+1} = 0.49U_{13,j} + 0.49U_{15,j} + 0.02U_{14,j}$

$U_{15,j} = 0.49U_{14,j} + 0.49U_{16j} + 0.02U_{15,j}$

$U_{16,j} = 0.49U_{15,j} + 0.49U_{17j} + 0.02U_{16,j}$

$U_{17,j} = 0.49U_{16,j} + 0.49U_{18j} + 0.02U_{17,j}$

$U_{18,j} = 0.49U_{17,j} + 0.49U_{19j} + 0.02U_{18,j}$

$U_{19,j} = 0.49U_{18,j} + 0.49U_{20j} + 0.02U_{19,j}$

For the boundary condition

$T(x,0) = 3x^2$ with x ranging from 0 to 6cm with step size of 0.3

~~$T(x,0) = 3x^2 = 3(0.3)^2 = 0.27$~~

~~$T(x_2$~~

$T(x_1,0) = 3x^2 = 3(0.3)^2 = 0.27$

$T(x_2,0) = 3x^2 = 3(0.6)^2 = 1.08$

$T(x_3,0) = 3x^2 = 3(1.2)^2 = 2.43$

$T(x_4,0) = 3x^2 = 3(1.5)^2 = 4.32$

$T(x_5,0) = 3x^2 = 3(1.8)^2 = 6.75$

$T(x_6,0) = 3x^2 = 3(2.1)^2 = 9.72$

$T(x_7,0) = 3x^2 = 3(2.4)^2 = 13.23$

$T(x_8,0) = 3x^2 = 3(2.7)^2 = 17.28$

$T(x_9,0) = 3x^2 = 3(3.1)^2 = 21.87$

$T(x_{10},0) = 3x^2 = 3(3)^2 = 27$

$T(x_{11},0) = 3x^2 = 3(3.6)^2 = 38.88$

$T(x_{12},0) = 3x^2 = 3(3.9)^2 = 45.63$

$T(x_{13},0) = 3x^2 = 3(4.2)^2 = 52.92$

$T(x_{14},0) = 3x^2 = 3(4.5)^2 = 60.75$

$T(x_{15},0) = 3x^2 = 3(4.8)^2 = 69.12$

$T(x_{16},0) = 3x^2 = 3(5.1)^2 = 78.03$

$T(x_{17},0) = 3x^2 = 3(5.4)^2 = 87.48$

$T(x_{18},0) = 3x^2 = 3(5.7)^2 = 94.74$

$T(x_{20},0) = 3x^2 = 3(6)^2 = 108$

$T(0,t) = 0, T(2,\bar{T}) = 108$

temperature has a range of 0 to 0.3hr with step size of 0.02hr To get to 0.02hr, $\bar{j} = 0$ when $j = 0$ (replacing U with T)

$T_{1,1} = 0.49U_{0,0} + 0.49U_{2,0} + 0.02U_{1,0}$

$= 0.49(0) + 0.49(1.08) + 0.02(0.27)$

$= 0.5346$

$\bar{T}_{2,1} = 0.49U_{1,0} + 0.49U_{3,0} + 0.02U_{2,0}$

$= 0.49(0.27) + 0.49(2.43) + 0.02(1.08)$

$= 1.3446$

$\bar{T}_{3,1} = 0.49U_{2,0} + 0.49U_{4,0} + 0.02U_{3,0}$

$= 0.49(1.08) + 0.49(4.32) + 0.02(2.43)$

$= 2.6976$

$\bar{T}_{4,1} = 0.49U_{3,0} + 0.49U_{5,0} + 0.02U_{4,0}$

$= 0.49(2.43) + 0.49(6.75) + 0.02(4.32)$

$= 4.5846$

$\bar{T}_{5,1} = 0.49U_{4,0} + 0.49U_{6,0} + 0.02U_{5,0}$

$= 0.49(7.32) + 0.49(9.72) + 0.02(0.75)$

$= 7.0146$

$\bar{T}_{6,1} = 0.49\bar{T}_{5,0} + 0.49\bar{T}_{7,0} + 0.02\bar{T}_{6,0}$

$= 0.49(6.75) + 0.49(13.23) + 0.02(9.72)$

$= 9.9846$

$\bar{T}_{7,1} = 0.49\bar{T}_{6,0} + 0.49\bar{T}_{8,0} + 0.02\bar{T}_{7,0}$

$= 0.49(9.72) + 0.49(17.27) + 0.02(13.23)$

$= 13.4946$

$\bar{T}_{8,1} = 0.49\bar{T}_{7,0} + 0.49\bar{T}_{9,0} + 0.02\bar{T}_{8,0}$

$= 0.49(13.23) + 0.49(21.87) + 0.02(17.28)$

$= 17.5446$

$\bar{T}_{9,1} = 0.49\bar{T}_{8,0} + 0.49\bar{T}_{10,0} + 0.02\bar{T}_{9,0}$

$= 0.49(17.28) + 0.49(22) + 0.02(21.87)$

$= 22.1346$

$\bar{T}_{10,1} = 0.49\bar{T}_{9,0} + 0.49\bar{T}_{11,0} + 0.02\bar{T}_{10,0}$

$= 0.49(21.87) + 0.49(32.67) + 0.02(27)$

$= 21.2646$

$\bar{T}_{11,1} = 0.49\bar{T}_{10,0} + 0.49\bar{T}_{12,0} + 0.02\bar{T}_{11,0}$

$= 0.49(29) + 0.49(38.88) + 0.02(32.67)$

$= 32.9346$

$\bar{T}_{12,1} = 0.49\bar{T}_{11,0} + 0.49\bar{T}_{13,0} + 0.02\bar{T}_{12,0}$

$= 0.49(32.67) + 0.49(45.63) + 0.02(38.88)$

$= 39.1446$

$T_{13,1} = 0.49\,T_{13,0} + 0.49\,T_{14,0} + 0.02\,T_{13,0}$

$= 0.49(38.88) + 0.49(59.2) + 0.02(45.63)$

$= 45.8946$

$T_{14,1} = 0.49\,T_{13,0} + 0.49\,T_{15,0} + 0.02\,T_{14,0}$

$= 0.49(45.63) + 0.49(60.75) + 0.02(52.92) = 53.1846$

$T_{15,1} = 0.49\,T_{14,0} + 0.49\,T_{16,0} + 0.02\,T_{15,0}$

$= 0.49(52.92) + 0.49(69.12) + 0.02(60.75) = 61.0146$

$T_{16,1} = 0.49\,T_{15,0} + 0.49\,T_{17,0} + 0.02\,T_{16,0}$

$= 0.49(60.75) + 0.49(78.03) + 0.02(69.12) = 69.3846$

$T_{17,1} = 0.49\,T_{16,0} + 0.49\,T_{18,0} + 0.02\,T_{17,0}$

$= 0.49(69.12) + 0.49(87.48) + 0.02(78.03)$

$= 78.2946$

$T_{18,1} = 0.49\,T_{17,0} + 0.49\,T_{19,0} + 0.02\,T_{18,0}$

$= 0.49(78.03) + 0.49(97.47) + 0.02(87.48)$

$= 87.7446$

$T_{19,1} = 0.49\,T_{18,0} + 0.49\,T_{20,0} + 0.02\,T_{19,0}$

$= 0.49(87.48) + 0.49(108) + 0.02(97.47)$

$= 97.7346$

Table for solving up to $t = 0.02$ hr and $x = 6$ cm

x	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0
$T(0)$	0	0.27	1.08	2.43	4.32	6.75	9.72	13.23	17.28	21.81	21.60
0.02	0	0.5346	1.3446	2.6946	4.5846	7.0146	9.9846	13.4946	17.5446	22.1346	27.2646

3.3	3.6	3.9	4.2	4.5	4.8	5.1	5.4	5.7	6.0
32.67	38.88	45.63	52.92	60.75	69.12	78.03	87.48	97.47	108
32.9346	39.1446	45.8946	53.1846	61.0146	69.3846	78.2946	87.7446	97.7346	108