EAG 382

ASSIGNMENT VII

1.) The model for the temperature distribution in a rod of length $L = 6\,cm$ is as given in eqn 1.

$$\frac{dT}{dt}(x,t) = \frac{d^2T(x,t)}{dx^2}$$

where $C = 2.2\,cm^2/hr$

with the conditions and the temperature (°C)

$$T(x,0) = 3x^2$$
$$T(0,t) = 0$$
$$T(L,t) = -10t.$$

Using $\Delta t = 0.02hr$ and $\Delta x = 0.3cm$, obtain the temperature profile of the system for $0 \leq t \leq 0.3hr$.

a.) Manually, in tabular form solving up to $t = 0.02hr$ and $x = 6cm$

Soln

Using Explicit forward Euler Method.

$$U_i^{k+1} = \gamma[U^k_{i+1} + U^k_{i-1}] + [1-2\gamma]U_i^k$$

when $i = 1$

$$U_{i,j+1} = \gamma U_{2,j} + \gamma U_{0j} + (1-2\gamma)U_{1,j}$$

but $\gamma = \frac{C \cdot dt}{(dx)^2} = \frac{2.2 \times 0.02}{0.3^2} = 0.49.$

$$[1 - (2 \times 0.49)] = 0.02$$

Rewriting the explicit forward Euler Method.

For when $i = 1$ to 19

$$U_{1,j+1} = 0.49U_{0,j} + 0.49U_{2,j} + 0.02U_{1,j}$$
$$U_{2,j+1} = 0.49U_{1,j} + 0.49U_{3,j} + 0.02U_{2,j}$$

$U_{3,j+1} = 0.49U_{2,j} + 0.49U_{4,j} + 0.02U_{3,j}$

$U_{4,j+1} = 0.49U_{3,j} + 0.49U_{5,j} + 0.02U_{4,j}$

$U_{5,j+1} = 0.49U_{4,j} + 0.49U_{6,j} + 0.02U_{5,j}$

$U_{6,j+1} = 0.49U_{5,j} + 0.49U_{7,j} + 0.02U_{6,j}$

$U_{7,j+1} = 0.49U_{6,j} + 0.49U_{8,j} + 0.02U_{7,j}$

$U_{8,j+1} = 0.49U_{7,j} + 0.49U_{9,j} + 0.02U_{8,j}$

$U_{9,j+1} = 0.49U_{8,j} + 0.49U_{10,j} + 0.02U_{9,j}$

$U_{10,j+1} = 0.49U_{9,j} + 0.49U_{11,j} + 0.02U_{10,j}$

$U_{11,j+1} = 0.49U_{10,j} + 0.49U_{12,j} + 0.02U_{11,j}$

$U_{12,j+1} = 0.49U_{11,j} + 0.49U_{13,j} + 0.02U_{12,j}$

$U_{13,j+1} = 0.49U_{12,j} + 0.49U_{14,j} + 0.02U_{13,j}$

$U_{14,j+1} = 0.49U_{13,j} + 0.49U_{15,j} + 0.02U_{14,j}$

$U_{15,j+1} = 0.49U_{14,j} + 0.49U_{16,j} + 0.02U_{15,j}$

$U_{16,j+1} = 0.49U_{15,j} + 0.49U_{17,j} + 0.02U_{16,j}$

$U_{17,j+1} = 0.49U_{16,j} + 0.49U_{18,j} + 0.02U_{17,j}$

$U_{18,j+1} = 0.49U_{17,j} + 0.49U_{19,j} + 0.02U_{18,j}$

$U_{19,j+1} = 0.49U_{18,j} + 0.49U_{20,j} + 0.02U_{19,j}$

For the boundary condition:-

$T(x,0) = 3x^2$ with x ranging from 0 to 6cm with step size of 0.3.

$T(x_1,0) = 3x^2 = 3(0.3)^2 = 0.27$

$T(x_2,0) = 3x^2 = 3(0.6)^2 = 1.08$

$T(x_3,0) = 3x^2 = 3(1.2)^2 = 2.43$

$T(x_4,0) = 3x^2 = 3(1.5)^2 = 4.32.$

$T(x_5,0) = 3x^2 = 3(1.8)^2 = 6.75$

$T(x_6,0) = 3x^2 = 3(2.1)^2 = 9.72.$

$T(x_7,0) = 3x^2 = 3(2.4)^2 = 13.23$

$T(x_8,0) = 3x^2 = 3(2.7)^2 = 17.28$

$T(x_9,0) = 3x^2 = 3(2.7)^2 = 21.87$

$T(x_{10},0) = 3x^2 = 3(3)^3 = 27$

$T(x_{10},0) = 3x^2 = 3(3)^2 = 32.67$

$T(x_{10},0) = 3x^2 = 3(3.6)^2 = 38.88$

$T(x_{13},0) = 3x^2 = 3(3.9)^2 = 45.63$

$T(x_{14},0) = 3x^2 = 3(4.2)^2 = 52.92.$

$T(x_{15},0) = 3x^2 = 3(4.5)^2 = 60.75.$

$T(x_{16},0) = 3x^2 = 3(4.8)^2 = 69.12.$

$T(x_{17},0) = 3x^2 = 3(5.1)^2 = 78.03$

$T(x_{18},0) = 3x^2 = 3(5.4)^2 = 87.48$

$T(x_{19},0) = 3x^2 = 3(5.7)^2 = 94.44$

$T(x_{20},0) = 3x^2 = 3(6)^2 = 108$

$T(0,t) = 0$, $T(L,t) = 108$.

temperature has a range of 0 to 0.3 hr with step size of 0.02 hr. To get to 0.02 hr, $j = 0$

When $j = 0$ [replacing u with T]

$T_{1,1} = 0.49u_{0,0} + 0.49u_{2,0} + 0.02u_{1,0}$
$= 0.49(0) + 0.49(1.08) + 0.02(0.27)$
$= 0.5346$

$T_{2,1} = 0.49u_{1,0} + 0.49u_{3,0} + 0.02u_{2,0}$
$= 0.49(0.27) + 0.49(2.48) + 0.02(1.08)$
$= 1.3446$

$T_{3,1} = 0.49u_{2,0} + 0.49u_{4,0} + 0.02u_{3,0}$
$= 0.49(1.08) + 0.49(4.32) + 0.02(2.43)$
$= 2.6946$

$T_{4,1} = 0.49u_{3,0} + 0.49u_{5,0} + 0.02u_{4,0}$
$= 0.49[2.43] + 0.49(6.75) + 0.02[4.32]$
$= 4.5846$

$T_{5,1} = 0.49u_{4,0} + 0.49u_{6,0} + 0.02u_{5,0}$
$= 0.49[4.32] + 0.49[9.72] + 0.02[6.75]$
$= 7.0146$

$T_{6,1} = 0.49T_{5,0} + 0.49T_{7,0} + 0.02T_{6,0}$
$= 0.49[6.75] + 0.49[13.23] + 0.02[9.72]$
$= 9.9846.$

$T_{7,1} = 0.49T_{6,0} + 0.49T_{8,0} + 0.02T_{7,0}$
$= 0.49[9.72] + 0.49[17.25] + 0.02[13.23]$
$= 13.4946$

$T_{8,1} = 0.49T_{7,0} + 0.49T_{9,0} + 0.02T_{8,0}$
$= 0.049(13.23) + 0.49[21.87] + 0.02[17.28]$
$= 17.5446$

$T_{9,1} = 0.49T_{8,0} + 0.49T_{10,0} + 0.02T_{9,0}$
$= 0.49[17.28] + 0.47[22] + 0.02[21.87]$
$= 22.1346$

$T_{10,1} = 0.49 T_{9,0} + 0.49 T_{11,0} + 0.02 T_{10,0}$

$= 0.49(21.87) + 0.49[32.67] + 0.02[27]$

$= 27.2646$

$T_{11,1} = 0.49 T_{10,0} + 0.49 T_{12,0} + 0.02 T_{11,0}$

$= 0.49(27) + 0.49(38.88) + 0.02[32.67]$

$= 32.9346$

$T_{12,1} = 0.49 T_{11,0} + 0.49 T_{13,0} + 0.02 T_{12,0}$

$= 0.49(32.67) + 0.49[45.63] + 0.02(38.88]$

$= 39.1446$

$T_{13,1} = 0.49 T_{12,0} + 0.49 T_{14,0} + 0.02 T_{13,0}$

$= 0.49(38.88) + 0.49(59.2) + 0.02[45.63] = 45.8946$

$T_{14,1} = 0.49 T_{13,0} + 0.49 T_{15,0} + 0.02 T_{14,0}$

$= 0.49(45.63) + 0.49(60.75) + 0.02[52.92]$

$= 53.1846$

$T_{15,1} = 0.49 T_{14,0} + 0.49 T_{16,0} + 0.02 T_{15,0}$

$= 0.49(52.92) + 0.49(69.12) + 0.02[60.75] = 61.0146$

$T_{16,1} = 0.49 T_{15,0} + 0.49 T_{17,0} + 0.02 T_{16,0}$

$= 0.49[60.75] + 0.49[78.03] + 0.02[69.12]$

$= 69.3846$

$T_{17,1} = 0.49 T_{16,0} + 0.49 T_{18,0} + 0.02 T_{17,0}$

$= 0.49[69.12] + 0.49[87.48] + 0.02[78.03]$

$= 78.2946$

$T_{18,1} = 0.49 T_{17,0} + 0.49 T_{19,0} + 0.02 T_{18,0}$

$= 0.49[78.03] + [0.49][97.47] + 0.02[87.48]$

$= 87.7446$

$T_{19,1} = 0.49 T_{18,0} + 0.49 T_{20,0} + 0.02 T_{19,0}$

$= 0.49(87.48] + 0.49[108] + 0.02[97.47]$

$= 97.7346$

x	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0	3.3
T_{co}	0	0.27	1.08	2.43	4.32	6.75	9.72	13.23	17.28	21.87	27.00	32.67
0.02	0	0.5346	1.3446	2.6946	4.5846	7.0146	9.9846	13.4946	17.4346	21.8746	27.2646	32.9346

x	3.9	4.2	4.5	4.8	5.1	5.4	5.7	6.0
T_{co}	45.63	52.92	60.75	69.12	78.03	87.48	97.47	108
0.02	45.8946	53.1846	61.0146	69.3846	78.2946	87.7446	97.7346	108