ASSIGNMENT VII

16/ENG02/034

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16/ENG02/034
Computer Engineering

QUESTION

The Model for the temperature distribution in a rod of length L 6m is as given in eqn (1)

$$\frac{\partial T(x,t)}{\partial t} = C \frac{\partial^2 T(x,t)}{\partial x^2}$$

where

$$C = 2.2 \frac{cm^2}{hr}$$

With the Conditions that the temp (°c):

$T(x,0) = 3x^2$

$T(0,t) = 0$

$T(L,t) = 108$

Using $\Delta t = 0.02$ hr and $\Delta x = 0.3$ cm, obtain the temp. Profile of the System @ $0 \leq t \leq 0.3$ hr

a) Manually, in tabular form, Solving up to $t = 0.02$ hr & $x = 6$ cm,

b) With the aid of Microsoft excel, in tabular & 3D graphical forms and

c) With the aid of MATLAB, in tabular & 3D graphical forms.

Solution

a) Using Explicit forward finite Method

$$u_i^{K+1} = r u_{i-1}^K + (1-2r) u_i^K + r u_{i+1}^K$$

$$r = C\frac{(\Delta t)}{(\Delta t)^2} = \frac{2.2 \times 0.02}{0.8^2}$$

$$r = 0.49$$

$$u_i^{K+1} = 0.49 u_{i-1,K} + 0.02 u_{i,K}^K + 0.49 u_{i+1,K}$$

for $i = 1$ to 19

1. $u_1^{K+1} = 0.49 u_{0,K} + 0.02 u_{1,K} + 0.49 u_{2,K}$
2. $u_2^{K+1} = 0.49 u_{1,K} + 0.02 u_{2,K} + 0.49 u_{3,K}$
3. $u_3^{K+1} = 0.49 u_{2,K} + 0.02 u_{3,K} + 0.49 u_{4,K}$
4. $u_4^{K+1} = 0.49 u_{3,K} + 0.02 u_{4,K} + 0.49 u_{5,K}$
5. $u_5^{K+1} = 0.49 u_{4,K} + 0.02 u_{5,K} + 0.49 u_{6,K}$
6. $u_6^{K+1} = 0.49 u_{5,K} + 0.02 u_{6,K} + 0.49 u_{4,K}$
7. $u_7^{K+1} = 0.49 u_{6,K} + 0.02 u_{4,K} + 0.49 u_{8,K}$
8. $u_8^{K+1} = 0.49 u_{4,K} + 0.02 u_{8,K} + 0.49 u_{9,K}$
9. $u_9^{K+1} = 0.49 u_{8,K} + 0.02 u_{9,K} + 0.49 u_{10,K}$
10. $u_{10}^{K+1} = 0.49 u_{9,K} + 0.02 u_{10,K} + 0.49 u_{11,K}$
11. $u_{11}^{K+1} = 0.49 u_{10,K} + 0.02 u_{11,K} + 0.49 u_{12,K}$
12. $u_{12}^{K+1} = 0.49 u_{11,K} + 0.02 u_{12,K} + 0.49 u_{13,K}$
13. $u_{13}^{K+1} = 0.49 u_{12,K} + 0.02 u_{13,K} + 0.49 u_{14,K}$
14. $u_{14}^{K+1} = 0.49 u_{13,K} + 0.02 u_{14,K} + 0.49 u_{15,K}$
15. $u_{15}^{K+1} = 0.49 u_{14,K} + 0.02 u_{15,K} + 0.49 u_{16,K}$
16. $u_{16}^{K+1} = 0.49 u_{15,K} + 0.02 u_{16,K} + 0.49 u_{17,K}$
17. $u_{17}^{K+1} = 0.49 u_{16,K} + 0.02 u_{17,K} + 0.49 u_{18,K}$
18. $u_{18}^{K+1} = 0.49 u_{17,K} + 0.02 u_{18,K} + 0.49 u_{19,K}$
19. $u_{19}^{K+1} = 0.49 u_{18,K} + 0.02 u_{19} + 0.49 u_{20,K}$

Boundary Condition

$T(x,0) = 3x^2$; $D(0.3)$ 6 cm

$T(x_1,0) = 3(0.3)^2 = 0.27$

$T(x_2,0) = 3(0.6)^2 = 1.08$

$T(x_3,0) = 3(0.9)^2 = 2.43$

$T(x_4,0) = 3(1.2)^2 = 4.32$

$T(x_5,0) = 3(1.5)^2 = 6.75$

$T(x_6,0) = 3(1.8)^2 = 9.72$

$T(x_7,0) = 3(2.1)^2 = 13.23$

$T(x_8,0) = 3(2.4)^2 = 17.28$

$T(x_9,0) = 3(2.7)^2 = 21.87$

$T(x_{10},0) = 3(3)^2 = 27$

$T(x_{11},0) = 3(3.3)^2 = 32.67$

$T(x_{12},0) = 3(3.6)^2 = 38.88$

$T(x_{13},0) = 3(3.9)^2 = 45.63$

$T(x_{14},0) = 3(4.2)^2 = 59.92$

$T(x_{15},0) = 3(4.5)^2 = 60.75$

$T(x_{16},0) = 3(4.8)^2 = 69.12$

$T(x_{17},0) = 3(5.1)^2 = 78.03$

$T(x_{18},0) = 3(5.4)^2 = 87.48$

$T(x_{19},0) = 3(5.7)^2 = 94.74$

$T(x_{20},0) = 3(6)^2 = 108$

$T(0,t) = 0$

$T(L,t) = 108$

Temperature range = 0 to 0.3 hr

$h = 0.02$ hr

For 0.02 hr, K = 0

$T_{1,1} = 0.49u_{0,0} + 0.49u_{2,0} + 0.02u_{1,0}$

$= 0.49(0) + 0.49(1.08) + 0.02(0.27) = 0.5346$

$T_{2,1} = 0.49u_{1,0} + 0.49u_{3,0} + 0.02u_{2,0}$

$= 0.49(0.27) + 0.49(2.43) + 0.02(1.08) = 1.3446$

$T_{3,1} = 0.49u_{2,0} + 0.49u_{4,0} + 0.02u_{3,0}$

$= 0.49(1.08) + 0.49(4.32) + 0.02(2.43) = 2.6946$

$T_{4,1} = 0.49u_{3,0} + 0.49u_{5,0} + 0.02u_{4,0}$

$= 0.49(2.43) + 0.49(6.75) + 0.02(4.32) = 4.5846$

$T_{5,1} = 0.49u_{4,0} + 0.49u_{6,0} + 0.02u_{5,0}$

$= 0.49(4.32) + 0.49(9.72) + 0.02(6.75) = 7.0146$

$T_{6,1} = 0.49u_{5,0} + 0.49u_{7,0} + 0.02u_{6,0}$

$= 0.49(6.75) + 0.49(13.23) + 0.02(9.72) = 9.9846$

$T_{7,1} = 0.49u_{6,0} + 0.49u_{8,0} + 0.02u_{7,0}$

$= 0.49(9.72) + 0.49(17.25) + 0.02(13.23) = 13.4946$

$T_{8,1} = 0.49u_{7,0} + 0.49u_{9,0} + 0.02u_{8,0}$

$= 0.49(13.28) + 0.49(21.87) + 0.02(17.28) = 17.5446$

$T_{9,1} = 0.49u_{8,0} + 0.49u_{10,0} + 0.02u_{9,0}$

$= 0.49(17.28) + 0.49(27) + 0.02(21.87) = 22.1346$

$T_{10,1} = 0.49u_{9,0} + 0.49u_{11,0} + 0.02u_{10,0}$

$= 0.49(21.87) + 0.49(32.67) + 0.02(27)$

$= 27.2646$

$T_{11,1} = 0.49u_{10,0} + 0.49u_{12,0} + 0.02u_{11,0}$

$= 0.49(27) + 0.49(38.88) + 0.02(32.67) = 32.9346$

$T_{12,1} = 0.49u_{11,0} + 0.49u_{13,0} + 0.02u_{12,0}$

$= 0.49(32.67) + 0.49(45.63) + 0.02(38.88) = 39.1446$

$T_{13,1} = 0.49(38.88) + 0.49(59.2) + 0.02(45.53) = 45.8946$

1 $T_{14,1} = 0.49u_{13,0} + 0.49u_{15,0} + 0.02u_{14,0}$

$= 0.49(45.63) + 0.49(69.12) + 0.02(60.75) = 53.1846$

$T_{15,1} = 0.49u_{14,0} + 0.49u_{16,0} + 0.02u_{15,0}$

$= 0.49(52.92) + 0.49(69.12) + 0.02(60.75) = 61.0146$

$T_{16,1} = 0.49u_{15,0} + 0.49u_{17,0} + 0.02u_{16,0}$

$= 0.49(60.75) + 0.49(78.03) + 0.02(69.12) = 69.3846$

$T_{17,1} = 0.49u_{16,0} + 0.49u_{18,0} + 0.02u_{17,0}$

$= 0.49(69.12) + 0.49(87.48) + 0.02(69.12) = 78.2946$

$T_{18,1} = 0.49u_{17,0} + 0.49u_{19,0} + 0.02u_{18,0}$

$= 0.49(78.03) + 0.49(97.49) + 0.02(87.48) = 87.7446$

$T_{19,1} = 0.49u_{18,0} + 0.49u_{20,0} + 0.02u_{19,0}$

$= 0.49(87.48) + 0.49(108) + 0.02(97.47) = 97.7346$

Table for solving up to $t = 0.02hr$ & $x = 6cm$

x	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4
T(0)	0	0.27	1.08	2.43	4.32	6.75	9.72	13.23	17.28
0.02	0	0.5346	1.3446	2.6946	4.5846	7.0146	9.98461	13.4946	17.5446

2.7	3.0	3.3	3.6	3.9	4.2	4.5	4.8	5.1
21.87	27.00	32.67	38.88	45.63	52.92	60.75	69.12	78.03
22.1346	27.2646	32.9346	39.1446	45.8946	53.1846	61.0146	69.3846	78.2946

5.4	5.7	6.0
87.48	97.47	108
87.7446	97.7346	108