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17/ENG021028
COMPUTER ENGINEERING

TEST
Question 4

A. Modelling is the mathematical representation and simulation of a system which involves solving the model and obtaining output variable for different values of input variable

B. Methods of obtaining Engineering system models

- i. Differentiating
- ii. Balance law
- iii. Analysis

Ci From Newton's Law of cooling

$$\frac{dT}{dt} = k(T - T_A) \quad \text{By separating variables}$$

$$\frac{dT}{(T - T_A)} = k dt \quad \text{Integrating through}$$

$$\int \frac{dT}{(T - T_A)} = \int k dt \quad \equiv \quad \ln(T - T_A) = kt + C$$

$$T - T_A = e^{kt+C} \quad \equiv \quad T - T_A = e^{kt} e^C$$

Initially $e^C = C$

T (initial reading of thermometer) = 10°C

T_A (Actual temperature of system) = 25°C at time $(t) = 0$

$$\therefore T - T_A = Ce^{kt}$$

$$10 - 25 = (C e^{k(0)})$$

$$-15 = C$$

After 3 minutes

$$T = 20 \quad t = 5$$

$$T - T_A = -15e^{kt}$$

$$20 - 25 = -15e^{k(5)}$$

$$\frac{-5}{-15} = e^{5k}$$

$$\ln(0.33) = 5k$$

$$k = \frac{-1.0986}{5}$$

$$k = -0.222$$

$$T - T_A = Ce^{kt}$$

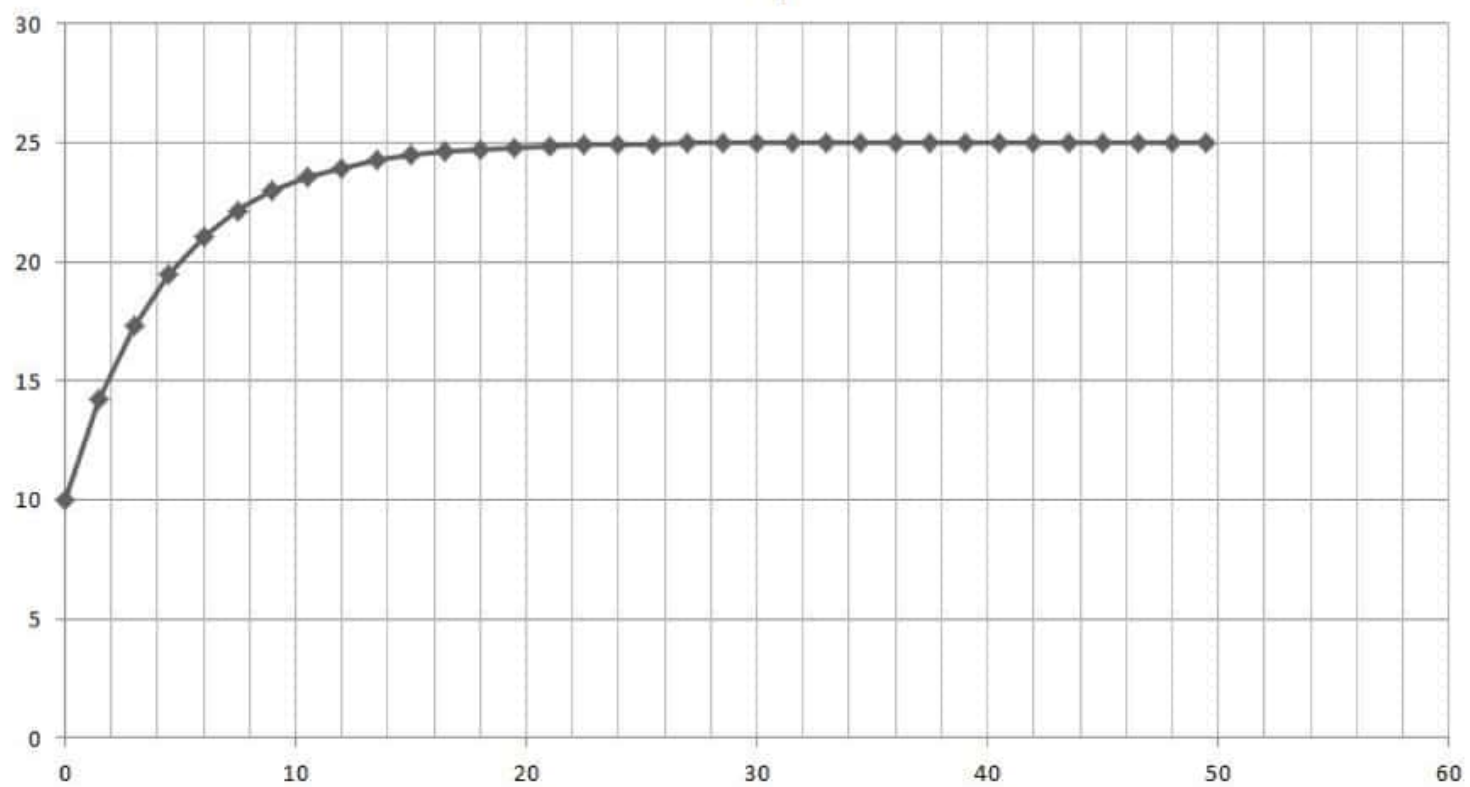
$$T = T_A + Ce^{kt}$$

$$T = 25 - 15e^{-0.222t}$$

ii Microsoft excel simulation

$$t = 1 \rightarrow 50 \quad \Delta t = 1.5 \quad T = 25 - 15 * (\text{EXP}(-0.222) * t)$$

T



iii MATLAB SIMULATION

Code

Command window

clear

clc

close all

syms t

t = 0:0.5:50

T = 25 - 15 * (exp(-0.222 * t))

Tb = subs(T)

plot(t, Tb)

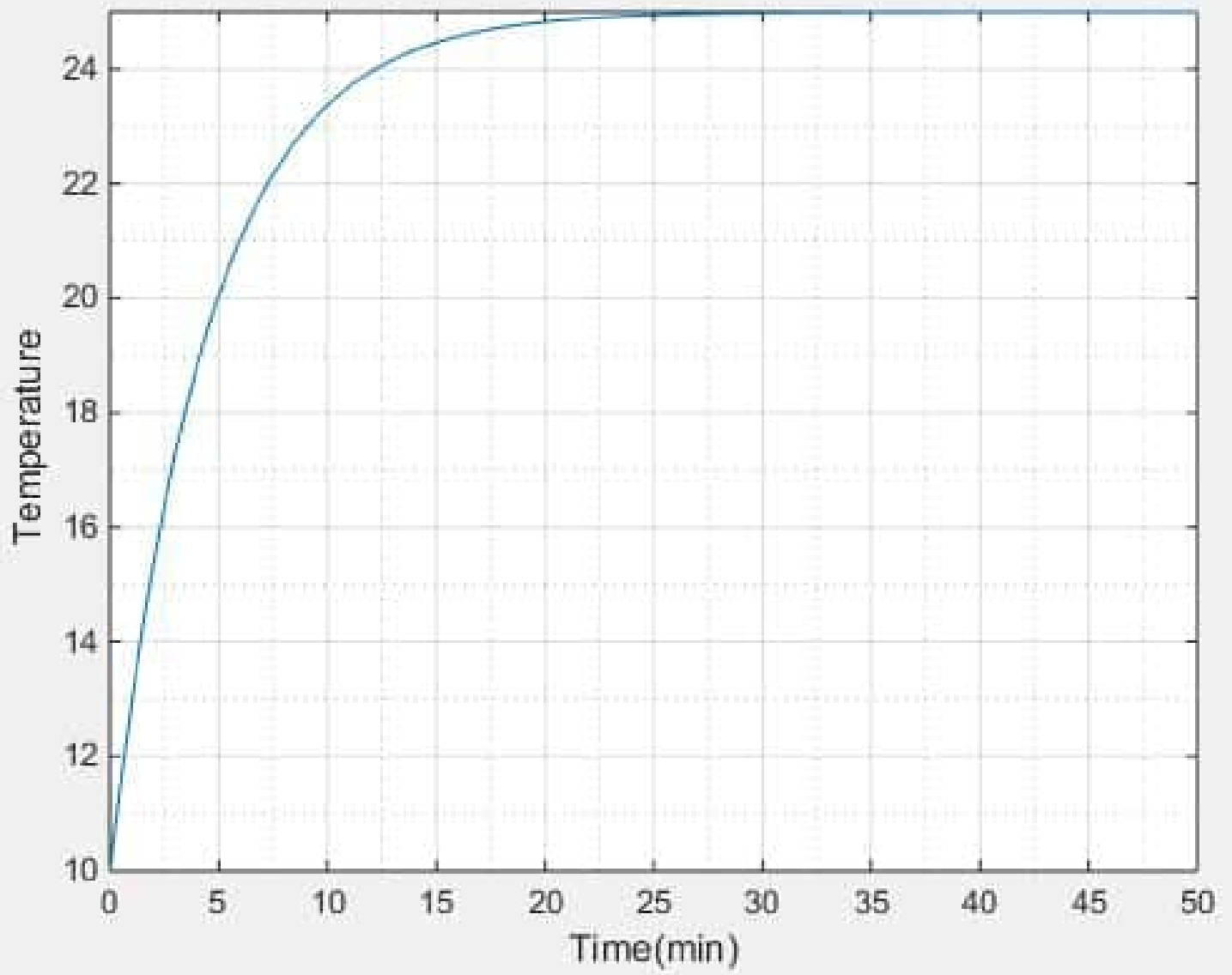
xlabel('Time(min)')

ylabel('Temperature')

axis tight

grid on

grid minor



iv. The steady state temperature of the system is 25°C

v. The system is not stable as the temperature changes with time

vi. We know that $\bar{T} = T_a + Ce^{kt}$
Here our $\bar{T} = 24.9^{\circ}\text{C}$

$$24.9^{\circ}\text{C} = 25 + 15e^{-0.222t}$$

$$24.9 - 25 = -15e^{-0.222t}$$

$$\ln\left(\frac{-0.1}{-15}\right) = -0.222t$$

$$t = \frac{-5.011}{-0.222}$$

$$t = 22.57 \approx 23 \text{ minutes}$$

\therefore The required time for the thermometer to practically reach the system temperature (24.9°C) is 22.57 mins