

NDUKA NNAEMEKA FRANCIS. F

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COMPUTER ENGINEERING
ENG 382

(1) The model for the temperature distribution in a rod of length $L = 6\text{cm}$ is as given in eqn;

$$\frac{dT}{dt}(x, t) = \frac{d^2T(x, t)}{dx^2}$$

where; $C = 2.2 \text{ cm}^2/\text{hr}$

with the conditions and the temperature ($^{\circ}\text{C}$)

$$T(0, 0) = 300^{\circ}\text{C}$$

$$T(0, t) = 0$$

$$T(L, t) = -108$$

using $\Delta T = 0.02\text{hr}$ and $\Delta x = 0.3\text{cm}$, obtain the temperature profile of the system for $0 \leq t \leq 0.3\text{hr}$

(2) Manually, in tabular form solving up to $t = 0.02\text{hr}$ $L = 6\text{cm}$

Solution

using explicit forward Euler method

$$U_T^{k+1} = Y \left[U_T^k + U_{T-1}^k \right] + (1-2Y) U_T^k$$

when $T = 1$

$$U_{1,j+1} = \gamma U_{1,j} + (1-\gamma) U_{1,j}$$

$$\text{but } \gamma = \frac{C \cdot \Delta t}{(\Delta x)^2} = \frac{2.2 \times 0.02}{0.3^2} = 0.49$$

$$(1 - (2 \times 0.49)) = 0.02$$

Rewriting the explicit forward Euler method,

For when $T = 1$ to 19

$$U_{1,j+1} = 0.49 U_{1,j} + 0.49 U_{2,j} + 0.02 U_{1,j}$$

$$U_{2,j+1} = 0.49 U_{1,j} + 0.49 U_{3,j} + 0.02 U_{2,j}$$

$$U_{3,j+1} = 0.49 U_{2,j} + 0.49 U_{4,j} + 0.02 U_{3,j}$$

$$U_{4,j+1} = 0.49 U_{3,j} + 0.49 U_{5,j} + 0.02 U_{4,j}$$

$$U_{5,j+1} = 0.49 U_{4,j} + 0.49 U_{6,j} + 0.02 U_{5,j}$$

$$U_{6,j+1} = 0.49 U_{5,j} + 0.49 U_{7,j} + 0.02 U_{6,j}$$

$$U_{7,j+1} = 0.49 U_{6,j} + 0.49 U_{8,j} + 0.02 U_{7,j}$$

$$\begin{aligned}
 U_{8,11} &= 0.49U_{7,11} + 0.49U_{9,11} + 0.02U_{8,10} \\
 U_{9,11} &= 0.49U_{8,11} + 0.49U_{10,11} + 0.02U_{9,10} \\
 U_{10,11} &= 0.49U_{9,11} + 0.49U_{11,11} + 0.02U_{10,10} \\
 U_{11,11} &= 0.49U_{10,11} + 0.49U_{12,11} + 0.02U_{11,10} \\
 U_{12,11} &= 0.49U_{11,11} + 0.49U_{13,11} + 0.02U_{12,10} \\
 U_{13,11} &= 0.49U_{12,11} + 0.49U_{14,11} + 0.02U_{13,10} \\
 U_{14,11} &= 0.49U_{13,11} + 0.49U_{15,11} + 0.02U_{14,10} \\
 U_{15,11} &= 0.49U_{14,11} + 0.49U_{16,11} + 0.02U_{15,10} \\
 U_{16,11} &= 0.49U_{15,11} + 0.49U_{17,11} + 0.02U_{16,10} \\
 U_{17,11} &= 0.49U_{16,11} + 0.49U_{18,11} + 0.02U_{17,10} \\
 U_{18,11} &= 0.49U_{17,11} + 0.49U_{19,11} + 0.02U_{18,10} \\
 U_{19,11} &= 0.49U_{18,11} + 0.49U_{20,11} + 0.02U_{19,10}
 \end{aligned}$$

For the boundary condition

$$T(x, 0) = 3x^2 \text{ with } x \text{ ranging from } 0 \text{ to } 6 \text{ cm with step size of } 0.3$$

$$T(x, 0) = 3x^2 = 3(0.3)^2 = 0.27$$

$T(x_1, 0) = 3x^2 = 3(0.3)^2 = 0.27$	$T(x_{11}, 0) = 3x^2 = 3(3.6)^2 = 38.88$
$T(x_2, 0) = 3x^2 = 3(0.6)^2 = 1.08$	$T(x_{12}, 0) = 3x^2 = 3(3.9)^2 = 45.63$
$T(x_3, 0) = 3x^2 = 3(1.2)^2 = 2.43$	$T(x_{13}, 0) = 3x^2 = 3(4.2)^2 = 52.91$
$T(x_4, 0) = 3x^2 = 3(1.5)^2 = 4.32$	$T(x_{14}, 0) = 3x^2 = 3(4.5)^2 = 60.75$
$T(x_5, 0) = 3x^2 = 3(1.8)^2 = 6.45$	$T(x_{15}, 0) = 3x^2 = 3(4.8)^2 = 69.12$
$T(x_6, 0) = 3x^2 = 3(2.1)^2 = 9.72$	$T(x_{16}, 0) = 3x^2 = 3(5.1)^2 = 78.03$
$T(x_7, 0) = 3x^2 = 3(2.4)^2 = 13.23$	$T(x_{17}, 0) = 3x^2 = 3(5.4)^2 = 87.48$
$T(x_8, 0) = 3x^2 = 3(2.7)^2 = 17.28$	$T(x_{18}, 0) = 3x^2 = 3(5.7)^2 = 94.74$
$T(x_9, 0) = 3x^2 = 3(3.0)^2 = 21.87$	$T(x_{19}, 0) = 3x^2 = 3(6.0)^2 = 108$
$T(x_{10}, 0) = 3x^2 = 3(3)^2 = 27$	

$$T(0, t) = 0, T(6, t) = 108$$

temperature has a range of 0 to 0.3 hr with step size of 0.02 hr to get to 0.02 hr, $t = 0$

when $s = 0$ (replacing U with T)

$$\begin{aligned}
 T_{1,1} &= 0.49U_{0,0} + 0.49U_{2,0} + 0.02U_{1,0} \\
 &= 0.49(0) + 0.49(1.08) + 0.02(0.27)
 \end{aligned}$$

$$= 0.5346$$

$$\begin{aligned}\bar{T}_{2,1} &= 0.49 U_{1,0} + 0.49 U_{3,0} + 0.02 U_{4,0} \\ &= 0.49(0.27) + 0.49(2.43) + 0.02(1.08) \\ &= 1.3446\end{aligned}$$

$$\begin{aligned}\bar{T}_{3,1} &= 0.49 U_{2,0} + 0.49 U_{4,0} + 0.02 U_{5,0} \\ &= 0.49(1.08) + 0.49(4.32) + 0.02(2.43) \\ &= 2.6976\end{aligned}$$

$$\begin{aligned}\bar{T}_{4,1} &= 0.49 U_{3,0} + 0.49 U_{5,0} + 0.02 U_{6,0} \\ &= 0.49(2.43) + 0.49(6.75) + 0.02(4.32) \\ &= 4.5846\end{aligned}$$

$$\begin{aligned}\bar{T}_{5,1} &= 0.49 U_{4,0} + 0.49 U_{6,0} + 0.02 U_{7,0} \\ &= 0.49(7.32) + 0.49(9.72) + 0.02(0.75) \\ &= 7.0146\end{aligned}$$

$$\begin{aligned}\bar{T}_{6,1} &= 0.49 \bar{T}_{5,0} + 0.49 \bar{T}_{7,0} + 0.02 \bar{T}_{8,0} \\ &= 0.49(6.75) + 0.49(13.23) + 0.02(9.72) \\ &= 9.9846\end{aligned}$$

$$\begin{aligned}\bar{T}_{7,1} &= 0.49 \bar{T}_{6,0} + 0.49 \bar{T}_{8,0} + 0.02 \bar{T}_{9,0} \\ &= 0.49(9.72) + 0.49(17.27) + 0.02(13.23) \\ &= 13.4946\end{aligned}$$

$$\begin{aligned}\bar{T}_{8,1} &= 0.49 \bar{T}_{7,0} + 0.49 \bar{T}_{9,0} + 0.02 \bar{T}_{10,0} \\ &= 0.49(13.23) + 0.49(21.87) + 0.02(17.28) \\ &= 17.5446\end{aligned}$$

$$\begin{aligned}\bar{T}_{9,1} &= 0.49 \bar{T}_{8,0} + 0.49 \bar{T}_{10,0} + 0.02 \bar{T}_{11,0} \\ &= 0.49(17.28) + 0.49(22) + 0.02(21.87) \\ &= 22.1346\end{aligned}$$

$$\begin{aligned}\bar{T}_{10,1} &= 0.49 \bar{T}_{9,0} + 0.49 \bar{T}_{11,0} + 0.02 \bar{T}_{12,0} \\ &= 0.49(21.87) + 0.49(32.67) + 0.02(22) \\ &= 21.2646\end{aligned}$$

$$\begin{aligned}\bar{T}_{11,1} &= 0.49 \bar{T}_{10,0} + 0.49 \bar{T}_{12,0} + 0.02 \bar{T}_{13,0} \\ &= 0.49(29) + 0.49(38.88) + 0.02(32.67) \\ &= 32.9346\end{aligned}$$

$$\begin{aligned}\bar{T}_{12,1} &= 0.49 \bar{T}_{11,0} + 0.49 \bar{T}_{13,0} + 0.02 \bar{T}_{14,0} \\ &= 0.49(32.67) + 0.49(45.63) + 0.02(38.88) \\ &= 39.1446\end{aligned}$$

$$\bar{T}_{13,0} = 0.49 \bar{T}_{12,0} + 0.49 \bar{T}_{14,0} + 0.02 \bar{T}_{15,0}$$

$$= 0.49(38.88) + 0.49(59.2) + 0.02(45.63)$$

$$= 45.8946$$

$$\bar{T}_{14,0} = 0.49 \bar{T}_{13,0} + 0.49 \bar{T}_{15,0} + 0.02 \bar{T}_{16,0}$$

$$= 0.49(45.63) + 0.49(60.75) + 0.02(52.92) = 53.1846$$

$$\bar{T}_{15,0} = 0.49 \bar{T}_{14,0} + 0.49 \bar{T}_{16,0} + 0.02 \bar{T}_{17,0}$$

$$= 0.49(52.92) + 0.49(69.12) + 0.02(60.75) = 61.0146$$

$$\bar{T}_{16,0} = 0.49 \bar{T}_{15,0} + 0.49 \bar{T}_{17,0} + 0.02 \bar{T}_{18,0}$$

$$= 0.49(60.75) + 0.49(78.03) + 0.02(69.12) = 69.3846$$

$$\bar{T}_{17,0} = 0.49 \bar{T}_{16,0} + 0.49 \bar{T}_{18,0} + 0.02 \bar{T}_{19,0}$$

$$= 0.49(69.12) + 0.49(87.48) + 0.02(78.03)$$

$$= 78.2946$$

$$\bar{T}_{18,0} = 0.49 \bar{T}_{17,0} + 0.49 \bar{T}_{19,0} + 0.02 \bar{T}_{20,0}$$

$$= 0.49(78.03) + 0.49(97.47) + 0.02(87.48)$$

$$= 87.7446$$

$$\bar{T}_{19,0} = 0.49 \bar{T}_{18,0} + 0.49 \bar{T}_{20,0} + 0.02 \bar{T}_{21,0}$$

$$= 0.49(87.48) + 0.49(108) + 0.02(97.47)$$

$$= 97.7346$$

Table for solving up to $t = 0.02 \text{ hr}$ and $x = 6 \text{ cm}$

x	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0
$T(0)$	0	0.27	1.08	2.43	4.32	6.75	9.72	13.23	17.28	21.81	21.60
0.02	0	0.5346	1.3446	2.6946	4.5846	7.0146	9.9846	13.4946	17.5446	22.1346	27.2646

3.3	3.6	3.9	4.2	4.5	4.8	5.1	5.4	5.7	6.0
32.67	38.88	45.63	52.92	60.75	69.12	78.03	87.48	97.47	108
32.9346	39.1446	45.8946	53.1846	61.0146	69.3846	78.2946	87.7446	97.7346	108