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Computer Engineering
ENG 382

① The model for the temperature distribution in a rod of length L = 6cm is as given in eqt 1;

$$\frac{dT(x,t)}{dt} = \frac{Cd^2T(x,t)}{dx^2}$$

Where; $C = 2.2cm^2/hr$

with the conditions and the temperature (°C)

$$T(x,0) = 3x^2$$
$$T(0,t) = 0$$
$$T(L,t) = -108$$

Using $\Delta T = 0.02hr$ and $\Delta x = 0.3cm$, obtain the temperature profile of the system for $0 \le t \le 0.3hr$

ⓐ Manually, in tabular form, solving up to t = 0.02hr and x = 6cm.

solution.

Using explicit forward euler method

$$U_i^{k+1} = r[U_{i+1}^k + U_{i-1}^k] + [1-2r]U_i^k$$

when i = 1

$$U_{1,j+1} = rU_{2,j} + rU_{0,j} + (1-2r)U_{1,j}$$

$$\text{but } r = \frac{C.dt}{(dx)^2} = \frac{2.2 \times 0.02}{0.3^2} = 0.49$$

$$[1 - (2 \times 0.49)] = 0.02$$

Rewriting the explicit forward euler method;

For when i = 1 to 19

$$U_{1,j+1} = 0.49U_{0,j} + 0.49U_{2,j} + 0.02U_{1,j}$$
$$U_{2,j+1} = 0.49U_{1,j} + 0.49U_{3,j} + 0.02U_{2,j}$$
$$U_{3,j+1} = 0.49U_{2,j} + 0.49U_{4,j} + 0.02U_{3,j}$$
$$U_{4,j+1} = 0.49U_{3,j} + 0.49U_{5,j} + 0.02U_{4,j}$$
$$U_{5,j+1} = 0.49U_{4,j} + 0.49U_{6,j} + 0.02U_{5,j}$$
$$U_{6,j+1} = 0.49U_{5,j} + 0.49U_{7,j} + 0.02U_{6,j}$$
$$U_{7,j+1} = 0.49U_{6,j} + 0.49U_{8,j} + 0.02U_{7,j}$$
$$U_{8,j+1} = 0.49U_{7,j} + 0.49U_{9,j} + 0.02U_{8,j}$$
$$U_{9,j+1} = 0.49U_{8,j} + 0.49U_{10,j} + 0.02U_{9,j}$$

$u_{10,j+1} = 0.49u_{9,j} + 0.49u_{11,j} + 0.02u_{10,j}$

$u_{11,j+1} = 0.49u_{10,j} + 0.49u_{12,j} + 0.02u_{11,j}$

$u_{12,j+1} = 0.49u_{11,j} + 0.49u_{13,j} + 0.02u_{12,j}$

$u_{13,j+1} = 0.49u_{12,j} + 0.49u_{14,j} + 0.02u_{13,j}$

$u_{14,j+1} = 0.49u_{13,j} + 0.49u_{15,j} + 0.02u_{14,j}$

$u_{15,j+1} = 0.49u_{14,j} + 0.49u_{16,j} + 0.02u_{15,j}$

$u_{16,j+1} = 0.49u_{15,j} + 0.49u_{17,j} + 0.02u_{16,j}$

$u_{17,j+1} = 0.49u_{16,j} + 0.49u_{18,j} + 0.02u_{17,j}$

$u_{18,j+1} = 0.49u_{17,j} + 0.49u_{19,j} + 0.02u_{18,j}$

$u_{19,j+1} = 0.49u_{18,j} + 0.49u_{20,j} + 0.02u_{19,j}$

For the boundary condition.

$T(x,0) = 3x^2$ with x ranging from 0 to 6 cm with step size of 0.3

$T(x,0) = 3x^2 = 3(0.3)^2 = 0.27$

$T(x_2,0) = 3x^2 = 3(0.6)^2 = 1.08$

$T(x_3,0) = 3x^2 = 3(1.2)^2 = 2.43$

$T(x_4,0) = 3x^2 = 3(1.5)^2 = 4.32$

$T(x_5,0) = 3x^2 = 3(1.8)^2 = 6.75$

$T(x_6,0) = 3x^2 = 3(2.1)^2 = 9.72$

$T(x_7,0) = 3x^2 = 3(2.4)^2 = 13.23$

$T(x_8,0) = 3x^2 = 3(2.7)^2 = 17.28$

$T(x_9,0) = 3x^2 = 3(3.1)^2 = 21.87$

$T(x_{10},0) = 3x^2 = 3(3)^2 = 27$

$T(x_{11},0) = 3x^2 = 3(3.6)^2 = 38.88$

$T(x_{13},0) = 3x^2 = 3(3.9)^2 = 45.63$

$T(x_{14},0) = 3x^2 = 3(4.2)^2 = 52.92$

$T(x_{15},0) = 3x^2 = 3(4.5)^2 = 60.75$

$T(x_{16},0) = 3x^2 = 3(4.8)^2 = 69.12$

$T(x_{17},0) = 3x^2 = 3(5.1)^2 = 78.03$

$T(x_{18},0) = 3x^2 = 3(5.4)^2 = 87.48$

$T(x_{19},0) = 3x^2 = 3(5.7)^2 = 94.74$

$T(x_{20},0) = 3x^2 = 3(6)^2 = 108$

$T(0,t) = 0, \quad T(L,T) = 108$

temperature has a range of 0 to 0.3hr with step size of 0.02hr. To get to 0.02hr, $j=0$.

When $j=0$ [replacing u with T]

$T_{1,1} = 0.49u_{0,0} + 0.49u_{2,0} + 0.02u_{1,0}$

$= 0.49(0) + 0.49(1.08) + 0.02(0.27)$

$= 0.5346$

$T_{2,1} = 0.49u_{1,0} + 0.49u_{3,0} + 0.02u_{2,0}$

$= 0.49(0.27) + 0.49(2.43) + 0.02(1.08)$

$= 1.3446$

$$T_{3,1} = 0.49u_{2,0} + 0.49u_{4,0} + 0.02u_{3,0}$$
$$= 0.49(1.08) + 0.49(4.32) + 0.02(2.43)$$
$$= 2.6946$$

$$T_{4,1} = 0.49u_{3,0} + 0.49u_{5,0} + 0.02u_{4,0}$$
$$= 0.49(2.43) + 0.49(6.75) + 0.02(4.32)$$
$$= 4.5846$$

$$T_{5,1} = 0.49u_{4,0} + 0.49u_{6,0} + 0.02u_{5,0}$$
$$= 0.49(7.32) + 0.49(9.72) + 0.02(6.75)$$
$$= 7.0146$$

$$T_{6,1} = 0.49T_{5,0} + 0.49T_{7,0} + 0.02T_{6,0}$$
$$= 0.49(6.75) + 0.49(13.23) + 0.02(9.72)$$
$$= 9.9846$$

$$T_{7,1} = 0.49T_{6,0} + 0.49T_{8,0} + 0.02T_{7,0}$$
$$= 0.49(9.72) + 0.49(17.25) + 0.02(13.23)$$
$$= 13.4946$$

$$T_{8,1} = 0.49T_{7,0} + 0.49T_{9,0} + 0.02T_{8,0}$$
$$= 0.49(13.23) + 0.49(21.87) + 0.02(17.28)$$
$$= 17.5446$$

$$T_{9,1} = 0.49T_{8,0} + 0.49T_{10,0} + 0.02T_{9,0}$$
$$= 0.49(17.28) + 0.49(22) + 0.02(21.87)$$
$$= 22.1346$$

$$T_{10,1} = 0.49T_{9,0} + 0.49T_{11,0} + 0.02T_{10,0}$$
$$= 0.49(21.87) + 0.49(32.67) + 0.02(27)$$
$$= 27.2646$$

$$T_{11,1} = 0.49T_{10,0} + 0.49T_{12,0} + 0.02T_{11,0}$$
$$= 0.49(27) + 0.49(38.88) + 0.02(32.67)$$
$$= 32.9346$$

$$T_{12,1} = 0.49T_{11,0} + 0.49T_{13,0} + 0.02T_{12,0}$$
$$= 0.49(32.67) + 0.49(45.63) + 0.02(38.88)$$
$$= 39.1446$$

$$T_{13,1} = 0.49T_{12,0} + 0.49T_{14,0} + 0.02T_{13,0}$$
$$= 0.49(38.88) + 0.49(59.2) + 0.02[45.63] = 45.8946$$

$$T_{14,1} = 0.49T_{13,0} + 0.49T_{15,0} + 0.02T_{14,0}$$
$$= 0.49(45.63) + 0.49(60.75) + 0.02(52.92) = 53.1846$$

$$T_{15,1} = 0.49T_{14,0} + 0.49T_{16,0} + 0.02T_{15,0}$$
$$= 0.49(52.92) + 0.49(69.12) + 0.02(60.75) = 61.0146$$

$$T_{16,1} = 0.49\,T_{15,0} + 0.49\,T_{17,0} + 0.02\,T_{16,0}$$
$$= 0.49(60.75) + 0.49(78.03) + 0.02(69.12) = 69.3846.$$

$$T_{17,1} = 0.49\,T_{16,0} + 0.49\,T_{18,0} + 0.02\,T_{17,0}$$
$$= 0.49(69.12) + 0.47(87.48) + 0.02(78.03)$$
$$= 78.2946.$$

$$T_{18,1} = 0.49\,T_{17,0} + 0.49\,T_{19,0} + 0.02\,T_{18,0}$$
$$= 0.49(78.03) + 0.49(97.47) + 0.02(87.48) = 87.7446.$$

$$T_{19,1} = 0.49\,T_{18,0} + 0.49\,T_{20,0} + 0.02\,T_{19,0}$$
$$= 0.49(87.48) + 0.49(108) + 0.02(97.47)$$
$$= 97.7346.$$

Table for solving up to $t = 0.02$ hr and $x = 6$ cm.

x	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0	3.3	3.6
T(0)	0	0.27	1.08	2.43	4.32	6.75	9.72	13.23	17.28	21.87	27.00	32.67	38.88
0.02	0	0.5346	1.3446	2.6946	4.5846	7.0146	9.98461	13.4946	17.5446	22.1346	27.2646	32.9346	39.1446

x	3.9	4.2	4.5	4.8	5.1	5.4	5.7	6.0
T(0)	45.63	52.92	60.75	69.12	78.03	87.48	97.47	108
0.02	45.8946	53.1846	61.0146	69.3846	78.2946	87.7446	97.7346	108