Question

The model for the temperature distribution in a rod of length $L = 6m$ is as given in the equation below

$$\frac{\partial T(x,t)}{\partial t} = C \frac{\partial^2 T(x,t)}{\partial x^2}$$

where

$$C = 2.2 \frac{cm^2}{hr}$$

with the conditions that the temperature (°C):

$$T(x,0) = 3x^2$$

$$T(0,t) = 0$$

$$T(L,t) = 100$$

using $\Delta t = 0.02hr$ and $\Delta x = 0.3cm$, obtain the temperature profile of the system for $0 \leq t \leq 0.3hr$

(a) manually, in tabular form solving up to $t = 0.02hr$ and $x = 6cm$

(b) with the aid of Microsoft Excel, in tabular and 3D graphical forms and

(c) with the aid of MATLAB, in tabular and 3D graphical forms

Answer

Using Explicit forward euler method

$$U_{i,j+1} = r U_{i-1,j} + (1-2r) U_{i,j} + r U_{i+1,j}$$

where

$$r = C \frac{\Delta t}{\Delta x^2} = \frac{2.2 \times 0.02}{0.3^2}$$

$$r = 0.49$$

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$U_{i,j+1} = 0.49U_{i-1,j} + 0.02U_{i,j} + 0.49U_{i+1,j}$

for the boundary condition

$T(x,0) = 3x^2$ with x running from 0 to 6cm with stepsize of 0.3

$T(x_0,0) = 3x^2 = 3(0.3)^2 = 0.27$

$T(x_1,0) = 3x^2 = 3(0.6)^2 = 1.08$

$T(x_2,0) = 3x^2 = 3(0.9)^2 = 2.43$

$T(x_3,0) = 3x^2 = 3(1.2)^2 = 4.32$

$T(x_4,0) = 3x^2 = 3(1.5)^2 = 6.75$

$T(x_5,0) = 3x^2 = 3(1.8)^2 = 9.72$

$T(x_6,0) = 3x^2 = 3(2.1)^2 = 13.23$

$T(x_7,0) = 3x^2 = 3(2.4)^2 = 17.28$

$T(x_8,0) = 3x^2 = 3(2.7)^2 = 21.87$

$T(x_9,0) = 3x^2 = 3(3)^2 = 27$

$T(x_{10},0) = 3x^2 = 3(3.3)^2 = 32.67$

$T(x_{11},0) = 3x^2 = 3(3.6)^2 = 38.88$

$T(x_{12},0) = 3x^2 = 3(3.9)^2 = 45.63$

$T(x_{13},0) = 3x^2 = 3(4.2)^2 = 52.92$

$T(x_{14},0) = 3x^2 = 3(4.5)^2 = 60.75$

$T(x_{15},0) = 3x^2 = 3(4.8)^2 = 69.12$

$T(x_{16},0) = 3x^2 = 3(5.1)^2 = 78.03$

$T(x_{17},0) = 3x^2 = 3(5.4)^2 = 87.48$

$T(x_{18},0) = 3x^2 = 3(5.7)^2 = 94.7436$

$T(x_{19},0) = 3x^2 = 3(6)^2 = 108$

when $i=1$ to when $i=19$

$U_{1,j+1} = 0.49U_{0,j} + 0.02U_{1,j} + 0.49U_{2,j}$

$U_{2,j+1} = 0.49U_{1,j} + 0.02U_{2,j} + 0.49U_{3,j}$

$U_{3,j+1} = 0.49U_{2,j} + 0.02U_{3,j} + 0.49U_{4,j}$

$U_{4,j+1} = 0.49U_{3,j} + 0.02U_{4,j} + 0.49U_{5,j}$

$U_{5,j+1} = 0.49U_{4,j} + 0.02U_{5,j} + 0.49U_{6,j}$

$U_{6,j+1} = 0.49U_{5,j} + 0.02U_{6,j} + 0.49U_{7,j}$

$U_{7,j+1} = 0.49U_{6,j} + 0.02U_{7,j} + 0.49U_{8,j}$

$U_{8,j+1} = 0.49U_{7,j} + 0.02U_{8,j} + 0.49U_{9,j}$

$U_{9,j+1} = 0.49U_{8,j} + 0.02U_{9,j} + 0.49U_{10,j}$

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$U_{10,j+1} = 0.49U_{9,j} + 0.02U_{9,j} + 0.49U_{10,j}$

$U_{11,j+1} = 0.49U_{10,j} + 0.02U_{10,j} + 0.49U_{12,j}$

$U_{12,j+1} = 0.49U_{11,j} + 0.02U_{12,j} + 0.49U_{12,j}$

$U_{13,j+1} = 0.49U_{12,j} + 0.02U_{12,j} + 0.49U_{14,j}$

$U_{14,j+1} = 0.49U_{13,j} + 0.02U_{14,j} + 0.49U_{15,j}$

$U_{15,j+1} = 0.49U_{14,j} + 0.02U_{15,j} + 0.49U_{16,j}$

$U_{16,j+1} = 0.49U_{15,j} + 0.02U_{16,j} + 0.49U_{17,j}$

$U_{17,j+1} = 0.49U_{16,j} + 0.02U_{17,j} + 0.49U_{18,j}$

$U_{18,j+1} = 0.49U_{17,j} + 0.02U_{18,j} + 0.49U_{19,j}$

$U_{19,j+1} = 0.49U_{18,j} + 0.02U_{19,j} + 0.49U_{20,j}$

$U_{20,j+1} = 0.49U_{19,j} + 0.02U_{20,j} + 0.49U_{21,j}$

To get for $t = 0.2$ hr

Solving for when $j = 0$ (replacing U with T)

when $j = 0$

$T_{1,1} = 0.49T_{0,0} + 0.02T_{1,0} + 0.49T_{2,0}$

$= 0.49(0) + 0.02(0.27) + 0.49(1.08)$

$= 0.5346$

$T_{2,1} = 0.49T_{1,0} + 0.02T_{2,0} + 0.49T_{3,0}$

$= 0.49(0.27) + 0.02(1.08) + 0.49(2.43)$

$= 1.3446$

$T_{3,1} = 0.49T_{2,0} + 0.02T_{3,0} + 0.49T_{4,0}$

$= 0.49(1.08) + 0.02(2.43) + 0.49(4.32)$

$= 2.6946$

$T_{4,1} = 0.49T_{3,0} + 0.02T_{4,0} + 0.49T_{5,0}$

$= 0.49(2.43) + 0.02(4.32) + 0.49(6.75)$

$= 4.5846$

$T_{5,1} = 0.49T_{4,0} + 0.02T_{5,0} + 0.49T_{6,0}$

$= 0.49(4.32) + 0.02(6.75) + 0.49(9.72)$

$= 7.0146$

$T_{6,1} = 0.49T_{5,0} + 0.02T_{6,0} + 0.49T_{7,0}$

$= 0.49(6.75) + 0.02(9.72) + 0.49(13.23)$

$= 9.9846$

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$\tau_{7,1} = 0.49\tau_{6,0} + 0.02\tau_{7,0} + 0.49\tau_{8,0}$

$= 0.49(9.72) + 0.49(17.28) + 0.02(13.23)$

$= 13.4946$

$\tau_{8,1} = 0.49\tau_{9,0} + 0.02\tau_{8,0} + 0.49\tau_{9,0}$

$= 0.49(13.23) + 0.02(17.28) + 0.49(21.87)$

$= 17.5446$

$\tau_{9,1} = 0.49\tau_{8,0} + 0.02\tau_{9,0} + 0.49\tau_{10,0}$

$= 0.49(17.28) + 0.02(21.87) + 0.49(27)$

$= 22.1346$

$\tau_{10,1} = 0.49\tau_{9,0} + 0.02\tau_{10,0} + 0.49\tau_{11,0}$

$= 0.49(21.87) + 0.02(27) + 0.49(32.67)$

$= 27.2646$

$\tau_{11,1} = 0.49\tau_{10,0} + 0.02\tau_{11,0} + 0.49\tau_{12,0}$

$= 0.49(27) + 0.02(32.67) + 0.49(38.88)$

$= 32.9346$

$\tau_{12,11} = 0.49\tau_{11,0} + 0.02\tau_{12,0} + 0.49\tau_{13,0}$

$= 0.49(32.67) + 0.02(38.88) + 0.49(45.63)$

$= 39.1446$

$\tau_{13,11} = 0.49\tau_{12,0} + 0.02\tau_{13,0} + 0.49\tau_{14,0}$

$= 0.49(38.88) + 0.02(45.63) + 0.49(59.42)$

$= 45.8946$

$\tau_{14,11} = 0.49\tau_{13,0} + 0.02\tau_{14,0} + 0.49\tau_{15,0}$

$= 0.49(45.63) + 0.02(52.92) + 0.49(60.75)$

$= 53.1846$

$\tau_{15,11} = 0.49\tau_{14,0} + 0.02\tau_{15,0} + 0.49\tau_{16,0}$

$= 0.49(52.92) + 0.02(60.75) + 0.49(69.12)$

$= 61.0146$

$\tau_{16,11} = 0.49\tau_{15,0} + 0.02\tau_{16,0} + 0.49\tau_{17,0}$

$= 0.49(60.75) + 0.02(69.12) + 0.49(78.03)$

$= 69.3846$

$\tau_{17,11} = 0.49\tau_{16,0} + 0.02\tau_{17,0} + 0.49\tau_{18,0}$

$= 0.49(69.12) + 0.02(78.03) + 0.49(87.42)$

$= 78.2946$

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$\tau_{16,11} = 0.49\tau_{15,0} + 0.02\tau_{16,0} + 0.49\tau_{17,0}$

$= 0.49(60.75) + 0.02(69.12) + 0.49(78.03)$

$= 69.3846$

$\tau_{17,11} = 0.49\tau_{16,0} + 0.02\tau_{17,0} + 0.49\tau_{18,0}$

$= 0.49(69.12) + 0.02(78.03) + 0.49(87.48)$

$= 78.2946$

$\tau_{18,1} = 0.49\tau_{17,1} + 0.02\tau_{18,0} + 0.49\tau_{19,0}$

$= 0.49(78.03) + 0.02(87.48) + 0.49(9$

$= 87.7446$

$\tau_{19,1} = 0.49\tau_{18,0} + 0.02\tau_{19,0} + 0.49\tau_{20,0}$

$= 0.49(87.48) + 0.02(97.47) + 0.49($

$= 97.7346$

i	z_i	$b z_0$	$b = 0.02$
0	0	0	0
1	0.3	0.21	0.5346
2	0.6	1.08	1.3446
3	0.9	2.43	2.6946
4	1.2	4.32	4.5846
5	1.5	6.75	7.0146
6	1.8	9.72	9.9846
7	2.1	13.23	13.4946
8	2.4	17.28	17.5446
9	2.7	21.87	22.1346
10	3	27	27.2646
11	3.3	32.67	32.9346
12	3.6	38.88	39.1446
13	3.9	45.63	45.8946
14	4.2	52.92	53.1846
15	4.5	60.75	61.0146
16	4.8	69.12	69.3846
17	5.1	78.03	78.2946
18	5.4	87.48	87.7446
19	5.7	94.7436	97.7346
20	60	108	108

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