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Elect/Elect

The model for the temperature distribution in a rod of length L = 6cm is as given in.

$$\frac{\partial T(x,t)}{\partial t} = C \frac{\partial^2 T(x,t)}{\partial x^2}$$

where $C = 2.2\ cm^2/hr$

with the conditions that the Temperature (°C);

$T(x,0) = 3x^2$

$T(0,t) = 0$

$T(l,t) = 108$

Using $\Delta t = 0.02hr$ and $\Delta x = 0.3\ cm$, obtain the temperature profile of the system for $0 \leq t \leq 0.3hr$.

(b) Manually in tabular form, solving up to $t = 0.02hr$ and $x = 6cm$.

Solution

Using Explicit forward Euler method

$$U_i^{x+1} = r(U_{i+1}^x + U_{i-1}^x) + (1 - 2r)U_i^x$$

when $i = 1$

$$U_{1,j+1} = rU_{2,j} + rU_{0,j} + (1 - 2r)U_{1,j}$$

but $r = \frac{C\,\Delta t}{(\Delta x)^2} = \frac{2.2 \times 0.02}{0.3^2}$

$r = 0.49$

$(1 - (2 \times 0.49)) = 0.02$

re writing the explicit forward euler method

for when i = 1 to 17

$U_{2,j+1} = 0.49U_{3,j} + 0.49U_{2,j} + 0.02U_{1,j}$

$U_{2,j+1} = 0.49U_{1,j} + 0.49U_{3,j} + 0.02U_{2,j}$

$U_{3,j+1} = 0.49U_{2,j} + 0.49U_{4,j} + 0.02U_{3,j}$

$U_{4,j+1} = 0.49U_{3,j} + 0.49U_{5,j} + 0.02U_{4,j}$

$U_{5,j+1} = 0.49U_{4,j} + 0.49U_{6,j} + 0.02U_{5,j}$

$U_{6,j+1} = 0.49U_{5,j} + 0.49U_{7,j} + 0.02U_{6,j}$

$U_{7,j+1} = 0.49U_{6,j} + 0.49U_{8,j} + 0.02U_{7,j}$

$U_{8,j+1} = 0.49U_{7,j} + 0.49U_{9,j} + 0.02U_{8,j}$

$U_{9,j+1} = 0.49U_{8,j} + 0.49U_{10,j} + 0.02U_{9,j}$

$U_{10,j+1} = 0.49U_{9,j} + 0.49U_{11,j} + 0.02U_{10,j}$

$U_{11,j+1} = 0.49U_{10,j} + 0.49U_{12,j} + 0.02U_{11,j}$

$U_{12,j+1} = 0.49U_{11,j} + 0.49U_{13,j} + 0.02U_{12,j}$

$U_{13,j+1} = 0.49U_{12,j} + 0.49U_{14,j} + 0.02U_{13,j}$

$U_{14,j+1} = 0.49U_{13,j} + 0.49U_{15,j} + 0.02U_{14,j}$

$U_{15,j+1} = 0.49U_{14,j} + 0.49U_{16,j} + 0.02U_{15,j}$

$U_{16,j+1} = 0.49U_{15,j} + 0.49U_{17,j} + 0.02U_{16,j}$

$U_{17,j+1} = 0.49U_{16,j} + 0.49U_{18,j} + 0.02U_{17,j}$

$U_{18,j+1} = 0.49U_{17,j} + 0.49U_{19,j} + 0.02U_{18,j}$

$U_{19,j+1} = 0.49U_{18,j} + 0.49U_{20,j} + 0.02U_{19,j}$

For the boundary Condition.

$T(x,0) = 3x^2$ ... x moving from 0 – 6.0m with step size of 0.3

$T(x,0) = 3x^2 = 3(0.3)^2 = 0.27$

$T(x_2,0) = 3x^2 = 3(0.6)^2 = 1.08$

$T(x_3,0) = 3x^2 = 3(0.9)^2 = 2.43$

$T(x_4,0) = 3x^2 = 3(1.2)^2 = 4.32$

$T(x_5,0) = 3x^2 = 3(1.5)^2 = 6.75$

$T(x_6,0) = 3x^2 = 3(1.8)^2 = 9.72$

$T(x_7,0) = 3x^2 = 3(2.1)^2 = 13.23$

$T(x_8,0) = 3x^2 = 3(2.4)^2 = 17.28$

$T(x_9,0) = 3x^2 = 3(2.7)^2 = 21.87$

$T(x_{10},0) = 3\times3.3^2 =$

$T(x_{11},0) = 3\times3.6^2$

$T(x_{12},0) = 3\times3.9^2$

$T(x_{13},0) = 3\times4.2^2$

$T(x_{14},0) = 3\times4.5^2$

$T(x_{15},0) = 3\times4.8^2$

$T(x_{16},0) = 3\times5.1^2$

$T(x_{17},0) = 3\times5.4^2$

x	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0	3.3	3.6	3.9	4.2	4.5	4.8
T																	
0	0	0.27	1.08	2.43	4.32	6.75	9.72	13.23	17.28	21.87	27	32.67	38.88	45.63	52.92	60.75	69.12
0.02	0	0.5346	1.3446	2.6946	4.5846	7.0146	9.8846	13.4946	17.5446	22.1346	27.2646	32.9346	39.1446	45.8946	53.1846	61.0146	69.3846

x	4.8	5.1	5.4	5.7	6.0
T					
0	69.12	78.03	87.48	97.47	108
0.02	69.3846	78.2946	87.7446	97.7346	108

Table for solving 0 to $t = 0.02hr$ and $x = 6m$

$T_{7,1} = 0.49\,T_{6,0} + 0.49\,T_{8,0} + 0.02\,T_{7,0}$

$= 0.49 \times 9.72 + 0.49(17.28) + 0.02(13.22)$

$= 13.4946$

$T_{11,1} = 0.49\,T_{7,0} + 0.49\,T_{12,0} + 0.02\,T_{11,0}$

$= 0.49 \times (13.22) + 0.49(21.87) + 0.02(17.28)$

$= 17.5446$

$T_{9,1} = 0.49\,T_{8,0} + 0.49\,T_{10,0} + 0.02\,T_{9,0}$

$= 0.49(17.28) + 0.49(22) + 0.20(21.87)$

$= 22.1346$

$T_{10,1} = 0.49\,T_{9,0} + 0.49\,T_{11,0} + 0.02\,T_{10,0}$

$= 0.49(21.87) + 0.49(32.69) + 0.02(27) = 27.2696$

$T_{11,1} = 0.49\,T_{10,0} + 0.49\,T_{12,0} + 0.02\,T_{11,0}$

$= 0.49(27) + 0.49(38.88) + 0.02(32.69) = 32.9396$

$T_{12,1} = 0.49\,T_{11,0} + 0.49\,T_{13,0} + 0.02\,T_{12,0}$

$= 0.49(32.69) + 0.49(45.63) + 0.02 \times 38.88 = 39.1946$

$T_{13,1} = 0.49\,T_{12,0} + 0.49\,T_{14,0} + 0.02\,T_{13,0}$

$= 0.49(38.88) + 0.49(37.2) + 0.02(45.63) = 43.2946$

$T_{14,1} = 0.49\,T_{13,0} + 0.49\,T_{15,0} + 0.02\,T_{14,0}$

$= 0.49(45.63) + 0.49(60.75) + 0.02(52.92) = 53.1846$

$T_{15,1} = 0.49\,T_{14,0} + 0.49(69.12) + 0.02(60.75)$

$= 0.49(52.92) + 0.49(69.12) + 0.02(60.75) = 61.0146$

$T_{16,1} = 0.49\,T_{15,0} + 0.49\,T_{17,0} + 0.02\,T_{16,0}$

$= 0.49(60.75) + 0.49(78.03) + 0.02(69.12)$

$= 69.3846$

$T_{17,1} = 0.49\,T_{16,0} + 0.49\,T_{18,0} + 0.02\,T_{17,0}$

$= 0.49(69.12) + 0.49(87.48) + 0.02(78.03) = 78.2946$

$T_{18,1} = 0.49\,T_{17,0} + 0.49\,T_{19,0} + 0.02\,T_{18,0}$

$= 0.49(78.03) + 0.49(97.47) + 0.02(87.48) = 87.7446$

$T_{19,1} = 0.49\,T_{18,0} + 0.49\,T_{20,0} + 0.02\,T_{19,0}$

$= 0.49(87.48) + 0.49(108) + 0.02(97.47)$

$= 97.2946$

$T(x_{10,0}) = 3x^1 = 3 \times 3^1 = 27$

$T(x_{11,0}) = 3x^2 = 3 \times 3.3^2 = 32.67$

$T(x_{12,0}) = 3x^2 = 3 \times 3.6^2 = 38.88$

$T(x_{13,0}) = 3x^2 = 3 \times 3.9^2 = 45.63$

$T(x_{14,0}) = 3x^2 = 3 \times 4.2^2 = 52.92$

$T(x_{15,0}) = 3x^2 = 3 \times 4.5^2 = 60.75$

$T(x_{16,0}) = 3x^2 = 3 \times 4.8^2 = 69.12$

$T(x_{17,0}) = 3x^2 = 3 \times 5.1^2 = 78.03$

$T(x_{18,0}) = 3x^2 = 3 \times 5.4^2 = 87.48$

$T(x_{19,0}) = 3x^2 = 3 \times 5.7^2 = 94.7436$

$T(x_{20,0}) = 3x^2 = 3 \times 6^2 = 108$

$T(0,6) = 0$ $\quad T(1,1) = 108$

When $j = 0$ (replacing u with T)

$T_{1,1} = 0.49T_{0,0} + 0.49T_{2,0} + 0.02T_{1,0}$

$= 0.49(0) + 0.49(1.08) + 0.02(0.27)$

$= 0.5346$

$T_{2,1} = 0.49T_{1,0} + 0.49T_{3,0} + 0.02T_{2,0}$

$= 0.49(0.27) + 0.49(3.43) + 0.02(1.08)$

$= 1.3446$

$T_{3,1} = 0.49T_{2,0} + 0.49T_{4,0} + 0.02T_{3,0}$

$= 0.49 \times (1.08) + 0.49(4.32) + 0.02(3.43)$

$= 2.6946$

$T_{4,1} = 0.49T_{3,0} + 0.49T_{5,0} + 0.02T_{4,0}$

$= 0.49 \times (3.43) + 0.49(6.75) + 0.02(4.32)$

$= 4.5846$

$T_{5,0} = 0.49T_{4,0} + 0.49T_{6,0} + 0.02T_{5,0}$

$= 0.49 \times (4.32) + 0.49(9.72) + 0.02(6.75)$

$= 2.1168 = 7.0146$

$T_{6,1} = 0.49T_{5,0} + 0.49T_{7,0} + 0.02T_{6,0}$

$0.49 \times (6.75) + 0.49(13.23) + 0.02(9.72)$

9.9846