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Elect/elect

Question 1 [20 Marks]

The model for the temperature distribution in a rod of length L = 6m is as given in Equation (1), $\frac{∂T(x,t)}{∂t}$ = $c\frac{∂^{2}T(x,t)}{∂x^{2}} $ ……………………………………….(1)

where

c = 2.2 cm2 /hr …………………………………………(2)

with the conditions that the temperature ($℃$):

T(x, 0) = 3x2…………………………………………………………. (3)

T(0, t) = 0 …………………………………………………………………(4)

T(L,t) = 108 ……………………………………………………………….(5)

Using ∆t= 02 .0 hr and ∆x =3.0 cm, obtain the temperature profile of the system for 0 ≤ t ≤ 3 .0 hr

1. manually, in tabular form, solving up to t =0 .02 hr and x = 6cm

SOLUTION

$\frac{∂T(x,t)}{∂t}$ = $c\frac{∂^{2}T(x,t)}{∂x^{2}}$

Simplifying the above equation using the forward difference method for first order and the central difference for the second order; gives:

$\frac{T\_{i+1,J}-T\_{i,j}}{∆t}$ = $c\frac{T\_{i,j+1 - 2T\_{i,j}+T\_{i,j-1}}}{(∆x)^{2}} $

 Ti+1,j – Ti,j  = $\frac{c∆t}{(∆x)^{2}}$ ( $T\_{i,j+1 - 2T\_{i,j}+T\_{i,j-1}}$)

Let r = = $\frac{c∆t}{(∆x)^{2}}$

Ti+1,j – Ti,j  = r ( $T\_{i,j+1 - 2T\_{i,j}+T\_{i,j-1}}$)

Ti+1,j  = r ( $T\_{i,j+1 - 2T\_{i,j}+T\_{i,j-1}}$) + Ti,j

Ti+1,j  = r $T\_{i,j+1+(1-2r) T\_{i,j}+rT\_{i,j-1}}$

The boundary conditions are:

T(0, t) = 0

T(L,t) = 108

And L = 6cm

T(x, 0) = 3x2

c = 2.2 cm2 /hr

$∆t$ = 0.02 hr

$∆x$ = 0.3cm

r = $\frac{c∆t}{(∆x)^{2}}$ = $\frac{2.2\*0.02}{0.3^{2}}$ = 0.4889

$ nx$ = $\frac{x}{∆x}$ = $\frac{6}{0.3}$ = 20

$nt$ = $\frac{t}{∆t}$ = $\frac{0.3}{0.02}$ = 15

|  |  |
| --- | --- |
|  | TIME t VALUES |
|  Distance x values |  | i |  0 | 1 |
| j |  | 0.00 | 0.02 |
| 0 | 0.0 | 0 | 0 |
| 1 | 0.3 | 0.27 | 0.534 |
| 2 | 0.6 | 1.08 | 1.344 |
| 3 | 0.9 | 2.43 | 2.694 |
| 4 | 1.2 | 4.32 | 4.584 |
| 5 | 1.5 | 6.75 | 7.014 |
| 6 | 1.8 | 9.72 | 9.984 |
| 7 | 2.1 | 13.23 | 13.494 |
| 8 | 2.4 | 17.28 | 17.544 |
| 9 | 2.7 | 21.87 | 22.134 |
| 10 | 3.0 | 27.00 | 27.264 |
| 11 | 3.3 | 32.67 | 32.934 |
| 12 | 3.6 | 38.88 | 39.144 |
| 13 | 3.9 | 45.63 | 45.894 |
| 14 | 4.2 | 52.92 | 53.184 |
| 15 | 4.5 | 60.75 | 61.014 |
| 16 | 4.8 | 69.12 | 69.384 |
| 17 | 5.1 | 78.03 | 78.294 |
| 18 | 5.4 | 87.48 | 87.744 |
| 19 | 5.7 | 97.47 | 97.734 |
| 20 | 6.0 | 108.00 | 108 |

1. with the aid of Microsoft Excel, in tabular and 3D graphical forms, and

SOLUTION

 



(c) with the aid of MATLAB, in tabular and 3D graphical forms.

SOLUTION