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16/ENG04/1034
Electrical Electronics Engineering
300 Level
ENG 382

The model for the temperature distribution in a rod of length, $L = 6cm$ is as given in equation 1:

$$\frac{dT}{dt}(x,t) = \frac{d^2T(x,t)}{dx^2}$$

where, $C = 2.2cm^2/hr$

with the conditions and the temperature (°C)

$T(x,0) = 3x^2$

$T(0,t) = 0$

$T(L,t) = -10t$

using $\Delta t = 0.02hr$ and $\Delta x = 0.3cm$, obtain the temperature profile of the system for $0 \leq t \leq 0.3hr$

a) manually, in tabular form, solving up to $t = 0.02hr$ & $x = 6cm$

Soln

Using explicit forward euler method;

for when $i = 1$ to 19

$u_{1,j+1} = 0.49u_{0,j} + 0.49u_{2,j} + 0.02u_{1,j}$

$u_{2,j+1} = 0.49u_{1,j} + 0.49u_{3,j} + 0.02u_{2,j}$

$u_{3,j+1} = 0.49u_{2,j} + 0.49u_{4,j} + 0.02u_{3,j}$

$u_{4,j+1} = 0.49u_{3,j} + 0.49u_{5,j} + 0.02u_{4,j}$

$u_{5,j+1} = 0.49u_{4,j} + 0.49u_{6,j} + 0.02u_{5,j}$

$u_{6,j+1} = 0.49u_{5,j} + 0.49u_{7,j} + 0.02u_{6,j}$

$u_{7,j+1} = 0.49u_{6,j} + 0.49u_{8,j} + 0.02u_{7,j}$

$u_{8,j+1} = 0.49u_{7,j} + 0.49u_{9,j} + 0.02u_{8,j}$

$u_{9,j+1} = 0.49u_{8,j} + 0.49u_{10,j} + 0.02u_{9,j}$

$u_{10,j+1} = 0.49u_{9,j} + 0.49u_{11,j} + 0.02u_{9,j}$

$U_{11,j+1} = 0.49U_{10,j} + 0.49U_{12j} + 0.02U_{10,j}$

$U_{12,j+1} = 0.49U_{11,j} + 0.49U_{13j} + 0.02U_{11,j}$

$U_{13,j+1} = 0.49U_{12,j} + 0.49U_{14,j} + 0.02U_{11,j}$

$U_{14,j+1} = 0.49U_{13,j} + 0.49U_{15,j} + 0.02U_{12,j}$

$U_{15,j+1} = 0.49U_{14,j} + 0.49U_{16,j} + 0.02U_{13,j}$

$U_{16,j+1} = 0.49U_{15,j} + 0.49U_{17,j} + 0.02U_{14,j}$

$U_{17,j+1} = 0.49U_{16,j} + 0.49U_{18,j} + 0.02U_{15,j}$

$U_{18,j+1} = 0.49U_{17,j} + 0.49U_{19,j} + 0.02U_{16,j}$

$U_{19,j+1} = 0.49U_{18,j} + 0.49U_{20,j} + 0.02U_{17,j}$

For the boarding condition

$T(x,0) = 3x^2$ with x ranging from 0 to 6cm with step size of 0.3.

$T(x_1,0) = 3x^2 = 3(0.3) = 0.27$

$T(x_2,0) = 1.08$

$T(x_3,0) = 2.43$

$T(x_4,0) = 4.32$

$T(x_5,0) = 6.75$

$T(x_6,0) = 9.72$

$T(x_7,0) = 13.23$

$T(x_8,0) = 17.28$

$T(x_9,0) = 21.87$

$T(x_{10},0) = 27$

$T(x_{11},0) = 38.88$

$T(x_{12},0) = 45.63$

$T(x_{13},0) = 52.92$

$T(x_{14},0) = 60.75$

$T(x_{15},0) = 69.12$

$T(x_{16},0) = 78.03$

$T(x_{17},0) = 84.48$

$T(x_{18},0) = 91.74$

$T(x_{19},0) = 98$

$T_{1,1} = 0.49u_0 + 0.49u_{2,0} + 0.02u_{1,0}$

$= 0.49(0) + 0.49(1.08) + 0.02(0.27)$

$= 0.5346$

$T_{2,1} = 1.3446$, $T_{3,1} = 2.6976$, $T_{4,1} = 4.5846$, $T_{5,1} = 7.0146$, $T_{6,1} = 9.9846$

$T_{7,1} = 13.7946$, $T_{8,1} = 17.5446$, $T_{9,1} = 22.1346$, $T_{10,1} = 27.2646$

$T_{11,1} = 32.9346$, $T_{12,1} = 39.1446$, $T_{13,1} = 45.8946$, $T_{14,1} = 53.1846$

$T_{15,1} = 61.0146$, $T_{16,1} = 69.3846$, $T_{17,1} = 78.2946$, $T_{18,1} = 87.7446$, $T_{19,1} = 97.7346$

Table for solving up to $t = 0.02$hr and $x = 6$cm

x	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0	3.3	3.6
$T(0)$	0	0.27	1.08	2.43	4.32	6.75	9.72	13.23	17.28	21.87	27.00	32.67	38.88
0.02	0	0.53	1.34	2.69	4.58	7.01	9.98	13.49	17.54	22.13	27.26	32.93	39.14

x	3.9	4.2	4.5	4.8	5.1	5.4	5.7	6.0
$T(0)$	45.63	52.92	60.75	69.12	78.03	87.48	97.47	108
0.02	45.89	53.18	61.014	69.38	78.29	87.74	97.73	108