

Solution - Manual

Question (a)

For finite difference form

$$\frac{T_{i,j+1} - T_{i,j}}{\Delta t} = C \times \left[\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{(\Delta x)^2} \right]$$

$$\text{where } C = \frac{2.2 \text{ cm}^2}{\text{hr}}$$

Making $T_{i,j+1} - T_{i,j}$ the subject of the formula and substituting for $C = \frac{2.2 \text{ cm}^2}{\text{hr}}$

$$\therefore T_{i,j+1} - T_{i,j} = C \frac{\Delta t}{(\Delta x)^2} \times [T_{i+1,j} - 2T_{i,j} + T_{i-1,j}]$$

$$\text{where } r = \frac{C \Delta t}{(\Delta x)^2} = \text{reckless number}$$

$$\text{Taking } C = 2.2 \text{ cm}^2/\text{hr}, \Delta t = 0.02 \text{ hr}, \Delta x = 0.3 \text{ cm}$$

$$r = \frac{2.2 \times (0.02)}{(0.3)^2} = 0.488889$$

$$r \approx 0.49$$

∴ substituting value for r into eqn (a),

$$T_{L,j+1} - T_{L,j} = 0.49 [T_{L+1,j} - 2T_{L,j} + T_{L-1,j}]$$

$$T_{L,j+1} = 0.49 T_{L+1,j} - 0.98 T_{L,j} + 0.49 T_{L-1,j}$$

$$T_{L,j+1} = 0.49 T_{L+1,j} + 0.02 T_{L,j} + 0.49 T_{L-1,j}$$

$$T_{L,j+1} = 0.49 T_{L+1,j} + (1 - 2 \times 0.49) T_{L,j} + 0.49 T_{L-1,j}$$

At $t(0) = 0 \text{ hr}$

$$T(x, 0) = 3x^2$$

\Rightarrow First Row

where $x = 0 \text{ cm}$

$$T(0, 0) = 3x^2 = 3 \times (0)^2 = 0 \text{ cm} = x_0$$

where $x = 0.3 \text{ cm}$

$$T(0.3, 0) = 3x^2 = 3 \times (0.3)^2 = 0.27 \text{ cm} = x_1$$

where $x = 0.6 \text{ cm}$

$$T(0.6, 0) = 3x^2 = 3 \times (0.6)^2 = 1.08 \text{ cm} = x_2$$

where $x = 0.9 \text{ cm}$

$$T(0.9, 0) = 3x^2 = 3 \times (0.9)^2 = 2.43 \text{ cm} = x_3$$

where $x = 1.2 \text{ cm}$

$$T(1.2, 0) = 3x^2 = 3 \times (1.2)^2 = 4.32 \text{ cm} = x_4$$

where $x = 1.5 \text{ cm}$

$$T(1.5, 0) = 3x^2 = 3 \times (1.5)^2 = 6.75 \text{ cm} = x_5$$

where $x = 1.8 \text{ cm}$

$$T(1.8, 0) = 3x^2 = 3 \times (1.8)^2 = 9.72 \text{ cm} = x_6$$

where $x = 2.1 \text{ cm}$

$$T(2.1, 0) = 3x^2 = 3 \times (2.1)^2 = 13.23 \text{ cm} = x_7$$

where $x = 2.4 \text{ cm}$
 $T(2.4, 0) = 3x^2 = 3 \times (2.4)^2 = 17.28 \text{ cm} = x_8$

where $x = 2.7 \text{ cm}$
 $T(2.7, 0) = 3x^2 = 3 \times (2.7)^2 = 21.87 \text{ cm} = x_9$

where $x = 3.0 \text{ cm}$
 $T(3.0, 0) = 3x^2 = 3 \times (3)^2 = 27 \text{ cm} = x_{10}$

where $x = 3.3 \text{ cm}$
 $T(3.3, 0) = 3x^2 = 3 \times (3.3)^2 = 32.67 = x_{11}$

where $x = 3.6 \text{ cm}$
 $T(3.6, 0) = 3x^2 = 3 \times (3.6)^2 = 38.88 \text{ cm} = x_{12}$

where $x = 3.9 \text{ cm}$
 $T(3.9, 0) = 3x^2 = 3 \times (3.9)^2 = 45.63 \text{ cm} = x_{13}$

where $x = 4.2 \text{ cm}$
 $T(4.2, 0) = 3x^2 = 3 \times (4.2)^2 = 52.92 \text{ cm} = x_{14}$

where $x = 4.5 \text{ cm}$
 $T(4.5, 0) = 3x^2 = 3 \times (4.5)^2 = 60.75 \text{ cm} = x_{15}$

where $x = 4.8 \text{ cm}$
 $T(4.8, 0) = 3x^2 = 3 \times (4.8)^2 = 69.12 \text{ cm} = x_{16}$

where $x = 5.1 \text{ cm}$
 $T(5.1, 0) = 3x^2 = 3 \times (5.1)^2 = 78.03 \text{ cm} = x_{17}$

where $x = 5.4 \text{ cm}$
 $T(5.4, 0) = 3x^2 = 3 \times (5.4)^2 = 87.48 \text{ cm} = x_{18}$

Where $\lambda = 5.7 \text{ cm}$

$$T(5.7, 0) = 3\lambda^2 = 3 \times (5.7)^2 = 97.47 \text{ cm} = \lambda_{17}$$

Where $\lambda = 6.0 \text{ cm}$

$$T(6.0, 0) = 3\lambda^2 = 3 \times (6.0)^2 = 108 \text{ cm} = \lambda_{20}$$

⇒ Second Row

$$T_{l,j+1} = 0.49 T_{l+1,j} + (1 - 2 \times 0.49) T_{l,j} + 0.49 T_{l-1,j}$$

$$\therefore T_{l,j+1} = 0.49 T_{l+1,j} + 0.02 T_{l,j} + 0.49 T_{l-1,j}$$

Where $l = 1, j = 0$

$$T_{1,1} = 0.49 T_{2,0} + 0.02 T_{1,0} + 0.49 T_{0,0}$$
$$= 0.49 \times (1.08) + 0.02(0.27) + 0.49(0)$$

$$T_{1,1} = 0.5346$$

Where $l = 2, j = 0$

$$T_{2,1} = 0.49 T_{3,0} + 0.02 T_{2,0} + 0.49 T_{1,0}$$
$$= 0.49 \times (2.43) + 0.02(1.08) + 0.49(0.27)$$

$$T_{2,1} = 1.3446$$

Where $l = 3, j = 0$

$$T_{3,1} = 0.49 T_{4,0} + 0.02 T_{3,0} + 0.49 T_{2,0}$$
$$= 0.49(4.32) + 0.02(2.43) + 0.49(1.08)$$

$$T_{3,1} = 2.6946$$

Where $l = 4, j = 0$

$$T_{4,1} = 0.49 T_{5,0} + 0.02 T_{4,0} + 0.49 T_{3,0}$$
$$= 0.49(6.75) + 0.02(4.32) + 0.49(2.43)$$

$$T_{4,1} = 4.5846$$

Where $l=5, j=0$

$$T_{5,1} = 0.49 T_{6,0} + 0.02 T_{5,0} + 0.49 T_{4,0}$$

$$T_{5,1} = 0.49(9.12) + 0.02(6.75) + 0.49(4.32)$$

$$T_{5,1} = 7.0146$$

Where $l=6, j=0$

$$T_{6,1} = 0.49 T_{7,0} + 0.02 T_{6,0} + 0.49 T_{5,0}$$

$$= 0.49(13.23) + 0.02(9.72) + 0.49(6.75)$$

$$T_{6,1} = 9.9846$$

Where $l=7, j=0$

$$T_{7,1} = 0.49 T_{8,0} + 0.02 T_{7,0} + 0.49 T_{6,0}$$

$$= 0.49(17.28) + 0.02(13.23) + 0.49(9.72)$$

$$T_{7,1} = 13.4946$$

Where $l=8, j=0$

$$T_{8,1} = 0.49 T_{9,0} + 0.02 T_{8,0} + 0.49 T_{7,0}$$

$$= 0.49(21.87) + 0.02(17.28) + 0.49(13.23)$$

$$T_{8,1} = 17.5446$$

Where $l=9, j=0$

$$T_{9,1} = 0.49 T_{10,0} + 0.02 T_{9,0} + 0.49 T_{8,0}$$

$$= 0.49(27) + 0.02(21.87) + 0.49(17.28)$$

$$T_{9,1} = 22.1346$$

Where $l=10, j=0$

$$T_{10,1} = 0.49 T_{11,0} + 0.02 T_{10,0} + 0.49 T_{9,0}$$

$$= 0.49(32.67) + 0.02(27) + 0.49(21.87)$$

$$T_{10,1} = 27.2646$$

Where $l=11, j=0$

$$T_{11,1} = 0.49 T_{12,0} + 0.02 T_{11,0} + 0.49 T_{10,0}$$

$$= 0.49(38.88) + 0.02(32.67) + 0.49(27)$$

$$T_{11,1} = 32.9346$$

Where $l=12, j=0$

$$\begin{aligned} T_{12,1} &= 0.49 T_{13,0} + 0.02 T_{12,0} + 0.49 T_{11,0} \\ &= 0.49(45.63) + 0.02(38.88) + 0.49(32.63) \\ &= 39.1446 \end{aligned}$$

Where $l=13, j=0$

$$\begin{aligned} T_{13,1} &= 0.49 T_{14,0} + 0.02 T_{13,0} + 0.49 T_{12,0} \\ &= 0.49(52.92) + 0.02(45.63) + 0.49(38.88) \\ &= 45.8946 \end{aligned}$$

Where $l=14, j=0$

$$\begin{aligned} T_{14,1} &= 0.49 T_{15,0} + 0.02 T_{14,0} + 0.49 T_{13,0} \\ &= 0.49(60.75) + 0.02(52.92) + 0.49(45.63) \\ &= 53.1846 \end{aligned}$$

Where $l=15, j=0$

$$\begin{aligned} T_{15,1} &= 0.49 T_{16,0} + 0.02 T_{15,0} + 0.49 T_{14,0} \\ &= 0.49(69.12) + 0.02(60.75) + 0.49(52.92) \\ &= 61.0146 \end{aligned}$$

Where $l=16, j=0$

$$\begin{aligned} T_{16,1} &= 0.49 T_{17,0} + 0.02 T_{16,0} + 0.49 T_{15,0} \\ &= 0.49(78.03) + 0.02(69.12) + 0.49(60.75) \\ &= 69.3846 \end{aligned}$$

Where $l=17, j=0$

$$\begin{aligned} T_{17,1} &= 0.49 T_{18,0} + 0.02 T_{17,0} + 0.49 T_{16,0} \\ &= 0.49(87.48) + 0.02(78.03) + 0.49(69.12) \\ &= 78.2946 \end{aligned}$$

Where $l=18, j=0$

$$\begin{aligned} T_{18,1} &= 0.49 T_{19,0} + 0.02 T_{18,0} + 0.49 T_{17,0} \\ &= 0.49(97.47) + 0.02(87.48) + 0.49(78.03) \\ &= 87.7446 \end{aligned}$$

Where $l = 19, j = 0$

$$T_{19,1} = 0.49 T_{20,0} + 0.02 T_{19,0} + 0.49 T_{18,0} \\ = 0.49(108) + 0.02(97.47) + 0.49(87.48)$$

$$T_{19,1} = 97.7346$$

$$T_{20,1} = 108$$

Table of values for $t = 0$ to $t = 0.02$ s and $x = 6$ cm

Time t	x = 6 cm																			
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0	3.3	3.6	3.9	4.2	4.5	4.8	5.1	5.4	5.7
0.01	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0	3.3	3.6	3.9	4.2	4.5	4.8	5.1	5.4	5.7	6.0
0.02	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0	3.3	3.6	3.9	4.2	4.5	4.8	5.1	5.4	5.7	6.0	6.3