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Department: MECHATRONICS

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Course Title: ENGR. MATHS TV

Assignment 7

$$\frac{dT(x,t)}{dt} = C \frac{d^2 T(x,t)}{dx^2}$$

$$L = 6\text{cm}$$

$$C = 2.2\text{cm}^2/\text{hr}$$

$$T(0,t) = 0$$

$$T(L,t) = 108 \quad \therefore T(6,t) = 108$$

$$T(x,0) = 3x^2$$

$$\Delta t = 0.02\text{hr} = k$$

$$\Delta x = 0.3\text{cm} = h$$

$$0 \leq t \leq 0.3\text{hr}$$

$$\text{recall } r = \frac{kC}{h^2} = \frac{0.02 \times 2.2}{0.3^2} = 0.4889 \approx 0.49$$

Recall general equation

$$U_{i,j+1} = rU_{i-1,j} + (1-2r)U_{i,j} + rU_{i+1,j}$$

at $t=0$

$$T(1,0) = 3x^2$$

$$= 3(0.3)^2 = 0.27$$

$$T(2,0) = 3x^2$$

$$= 3(0.6)^2 = 1.08$$

$$T(3,0) = 3x^2$$

$$= 3(0.9)^2 = 2.43$$

$$T(4,0) = 3x^2$$

$$= 3(1.2)^2 = 4.32$$

$$T(5,0) = 3x^2$$

$$= 3(1.5)^2 = 6.75$$

$$T(6,0) = 3x^2$$

$$= 3(1.8)^2 = 9.72$$

$$T(7,0) = 3x^2$$

$$= 3(2.1)^2 = 13.23$$

$$T(8,0) = 3x^2$$

$$= 3(2.4)^2 = 17.28$$

$$T(9,0) = 3x^2$$

$$= 3(2.7)^2 = 21.87$$

$$T(10,0) = 3x^2$$

$$= 3(3.0)^2 = 27$$

$$T(11,0) = 3x^2$$

$$= 3(3.3)^2 = 32.67$$

$$T(12,0) = 3x^2$$

$$= 3(3.6)^2 = 38.88$$

$$T(13,0) = 3x^2$$

$$= 3(3.9)^2 = 45.63$$

$$T(14,0) = 3x^2$$

$$= 3(4.2)^2 = 52.92$$

$$T(15,0) = 3x^2$$

$$= 3(4.5)^2 = 60.75$$

$$T(16,0) = 3x^2$$

$$= 3(4.8)^2 = 69.12$$

$$T(17,0) = 3x^2$$

$$= 3(5.1)^2 = 78.03$$

$$T(18,0) = 3x^2$$

$$= 3(5.4)^2 = 87.48$$

$$T(19,0) = 3x^2$$

$$= 3(5.7)^2 = 97.47$$

$$T(20,0) = T(4,T) = 108.$$

At $t = 1$

using $U_{i,j+1} = rU_{i-1,j} + (1-r)U_{i,j} + rU_{i+1,j}$

$$T_{i,j+1} = rT_{i-1,j} + (1-r)T_{i,j} + rT_{i+1,j}$$

at $i=0$

$$T_{(0,t)} = 0$$

at $i=1$

$$\begin{aligned} T_{1,t} &= rT_{0,0} + (1-2r)T_{1,0} + rT_{2,0} \\ &= 0.49(0.5) + (1-2(0.49))(0.27) + (0.49)(1.08) \\ &= 0.534 \end{aligned}$$

at $i=2$

$$\begin{aligned} T_{2,t} &= rT_{1,0} + (1-2r)T_{2,0} + rT_{3,0} \\ &= 0.49(0.27) + (1-2(0.49))(1.08) + 0.49(2.43) \\ &= 1.344 \end{aligned}$$

at $i=3$

$$\begin{aligned} T_{3,t} &= rT_{2,0} + (1-2r)T_{3,0} + rT_{4,0} \\ &= 0.49(1.08) + (1-2(0.49))(2.43) + 0.49(4.32) \\ &= 2.694 \end{aligned}$$

at $i=4$

$$\begin{aligned} T_{4,t} &= rT_{3,0} + (1-2r)T_{4,0} + rT_{5,0} \\ &= 0.49(2.43) + (1-2(0.49))(4.32) + 0.49(6.75) \\ &= 4.584 \end{aligned}$$

at $i=5$

$$\begin{aligned} T_{5,t} &= rT_{4,0} + (1-2r)T_{5,0} + rT_{6,0} \\ &= 0.49(4.32) + [1-2(0.49)](6.75) + 0.49(9.72) \\ &= 6.75 \end{aligned}$$

at $i=6$

$$\begin{aligned} T_{6,t} &= rT_{5,0} + (1-2r)T_{6,0} + rT_{7,0} \\ &= 0.49(6.75) + [1-2(0.49)](9.72) + 0.49(13.23) \\ &= 9.984 \end{aligned}$$

at $i=7$

$$\begin{aligned} T_{7,t} &= rT_{6,0} + (1-2r)T_{7,0} + rT_{8,0} \\ &= 0.49(9.72) + [1-2(0.49)](13.23) + 0.49(17.28) \\ &= 13.494 \end{aligned}$$

at $i=8$

$$\begin{aligned} T_{8,t} &= rT_{7,0} + (1-2r)T_{8,0} + rT_{9,0} \\ &= 0.49(13.23) + [1-2(0.49)](17.28) + 0.49(21.87) \\ &= 17.544 \end{aligned}$$

at $i=9$

$$\begin{aligned}T_{9,1} &= rT_{8,0} + (1-2r)T_{9,0} + rT_{10,0} \\&= 0.49(17.28) + [1-2(0.49)](21.87) + 0.49(27) \\&= 22.134\end{aligned}$$

at $i=10$

$$\begin{aligned}T_{10,1} &= rT_{9,0} + (1-2r)T_{10,0} + rT_{11,0} \\&= 0.49(21.87) + [1-2(0.49)](27) + 0.49(32.67) \\&= 27.264\end{aligned}$$

at $i=11$

$$\begin{aligned}T_{11,1} &= rT_{10,0} + (1-2r)T_{11,0} + rT_{12,0} \\&= 0.49(27) + [1-2(0.49)](32.67) + 0.49(38.88) \\&= 32.934\end{aligned}$$

at $i=12$

$$\begin{aligned}T_{12,1} &= rT_{11,0} + (1-2r)T_{12,0} + rT_{13,0} \\&= 0.49(32.67) + [1-2(0.49)](38.88) + 0.49(45.63) \\&= 39.144\end{aligned}$$

at $i=13$

$$\begin{aligned}T_{13,1} &= rT_{12,0} + (1-2r)T_{13,0} + rT_{14,0} \\&= 0.49(38.88) + [1-2(0.49)](45.63) + 0.49(52.72) \\&= 45.894\end{aligned}$$

at $i=14$

$$\begin{aligned}T_{14,1} &= rT_{13,0} + (1-2r)T_{14,0} + rT_{15,0} \\&= 0.49(45.63) + [1-2(0.49)](52.72) + 0.49(60.75) \\&= 53.184\end{aligned}$$

at $i=15$

$$\begin{aligned}T_{15,1} &= rT_{14,0} + (1-2r)T_{15,0} + rT_{16,0} \\&= 0.49(52.72) + [1-2(0.49)](60.75) + (0.49)(67.12) \\&= 61.014\end{aligned}$$

at $i=16$

$$\begin{aligned}T_{16,1} &= rT_{15,0} + (1-2r)T_{16,0} + rT_{17,0} \\&= 0.49(60.75) + [1-2(0.49)](67.12) + 0.49(78.03) \\&= 69.384\end{aligned}$$

at $i=17$

$$\begin{aligned} T_{17,1} &= rT_{16,0} + (1-2r)T_{17,0} + rT_{18,0} \\ &= 0.49(69.12) + [1-2(0.49)](78.08) + 0.49(87.48) \\ &= 78.294 \end{aligned}$$

at $i=18$

$$\begin{aligned} T_{18,1} &= rT_{17,0} + (1-2r)T_{18,0} + rT_{19,0} \\ &= 0.49(78.08) + [1-2(0.49)](87.48) + 0.49(97.47) \\ &= 87.744 \end{aligned}$$

at $i=19$

$$\begin{aligned} T_{19,1} &= rT_{18,0} + (1-2r)T_{19,0} + rT_{20,0} \\ &= 0.49(87.48) + [1-2(0.49)](97.47) + 0.49(108) \\ &= 97.734 \end{aligned}$$

at $i=20$

$$T_{20,1} = 108$$

$T_{(2k)}$	i	x																				
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
t	j																					
	0	0	0.27	1.08	2.43	4.32	6.75	9.72	13.20	17.28	21.96	27.36	32.67	38.88	45.00	52.02	60.00	69.12	78.08	87.48	97.47	108
	1	0	0.534	1.344	2.694	4.594	6.75	9.84	13.44	17.54	22.14	27.24	32.94	39.14	45.84	53.14	61.04	69.34	78.24	87.74	97.734	108
	2																					
	3																					