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16/ENG03/1032
Civil Engineering
ENG 382

Assignment 4

The model for the temperature distribution in a rod of leng
Le bar is given as eqn

$$\frac{dT(x,t)}{dt} = \frac{d^2T(x,t)}{dx}$$

Where C_2 2.2cm²/hr

with the conditions and temperature (°c)

$$T(x,0) = 3x^2$$

$$T(0,t) = 0$$

$$T(L,t) = -108$$

Using $\Delta t = 0.02$hr and $\Delta x = 0.3$cm, obtain the temperature Profil of the System for $0 \le t \le 0.3$hr

a manually in tabular form solving up to $t = 0.02$hr and $x =$ bar

Soln

Using explicit forward euler method

$$U_i^{k+1} = \alpha [U_{i+1}^k + U_{i-1}^k] + [1-2\alpha] U_i^k$$

When $i = 1$

$$U_{i,j+1} = \alpha U_{i,j} + \alpha U_{ij} + (1-2\alpha)U_{ij}$$

$$but\ \alpha = \frac{C * dt}{(dx)^2} = \frac{2.2 \times 0.02}{0.3^2} = 0.49$$

$$(1 - (C_2 * 0.49) = 0.02$$

for when $i = 1$ to 19

$$U_{i,j+1} = 0.49U_{0j} + 0.49U_{ij} + 0.02U_{ij}$$
$$U_{2,j+1} = 0.49U_{ij} + 0.49U_{3j} + 0.02U_{2j}$$
$$U_{2,j+1} = 0.49U_{2j} + 0.49U_{4j} + 0.02U_{3j}$$
$$U_{3,j+1} = 0.49U_{3j} + 0.49U_{5j} + 0.02U_{4j}$$
$$= 0.49U_{4j} + 0.49U_{6j} + 0.02U_{5j}$$

$U_{6,j+1} = 0.49U_{5j} + 0.49U_{7j} + 0.02U_{6,j}$

$U_{7,j+1} = 0.49U_{6j} + 0.49U_{8j} + 0.02U_{7,j}$

$U_{8,j+1} = 0.49U_{7j} + 0.49U_{9j} + 0.02U_{8,j}$

$U_{9,j+1} = 0.49U_{8j} + 0.49U_{10j} + 0.02U_{9,j}$

$U_{10,j+1} = 0.49U_{9j} + 0.49U_{11j} + 0.02U_{10j}$

$U_{11,j+1} = 0.49U_{10j} + 0.49U_{12j} + 0.02U_{11j}$

$U_{12,j+1} = 0.49U_{11j} + 0.49U_{13j} + 0.02U_{12j}$

$U_{13,j+1} = 0.49U_{12j} + 0.49U_{14j} + 0.02U_{13j}$

$U_{14,j+1} = 0.49U_{13j} + 0.49U_{15j} + 0.02U_{14j}$

$U_{15,j+1} = 0.49U_{14j} + 0.49U_{16j} + 0.02U_{15j}$

$U_{16,j+1} = 0.49U_{15j} + 0.49U_{17j} + 0.02U_{16j}$

$U_{17,j+1} = 0.49U_{16j} + 0.49U_{18j} + 0.02U_{17j}$

$U_{18,j+1} = 0.49U_{17j} + 0.49U_{19j} + 0.02U_{18j}$

$U_{19,j+1} = 0.49U_{18j} + 0.49U_{20j} + 0.02U_{19j}$

for the boundary Condition

$T(x,0) = 3x^2 = 3(0.3)^2 = 0.27$

$T(x_2\,0) = 3(0.6)^2 = 1.08$

$T(x_3\,0) = 3(0.2)^2 = 2.43$

$T(x_4\,0) = 3(1.2)^2 = 4.32$

$T(x_5\,0) = 3(1.5)^2 = 6.75$

$T(x_6\,0) = 3(1.8)^2 = 9.72$

$T(x_7\,0) = 3(2.1)^2 = 13.23$

$T(x_8\,0) = 3(2.4)^2 = 17.28$

$T(x_9\,0) = 3(2.7)^2 = 21.87$

$T(x_{10}\,0) = 3(3)^2 = 27$

$T(x_{10}\,0) = 3(3.3)^2 = 32.67$

$T(x_{11}\,0) = 3(3.6)^2 = 38.88$

$T(x_{12}\,0) = 3(3.9)^2 = 45.63$

$T(x_{14},0) = 3(4.2)^2 = 52.92$

$T(x_{15},0) = 3(4.5)^2 = 60.75$

$T(x_{16}\,0) = 3(4.8)^2 = 69.12$

$T(x_{17}\,0) = 3(5.1)^2 = 98.03$

$T(x_{18}\,0) = 3(5.4)^2 = 89.48$

$T(x_{19},0) = 3(5.7)^2 = 94.74$

$T(x_{20},0) = 3(6)^2 = 108$

$T(0,t) = 0$ $\quad T(L,t) = 108$

temperature has arrage of 0 to 0.3hr with step size of 0.02hr. To

When J = 0 [Replacing v with T]

$T_{1,1} = 0.49u_{0,6} + 0.49u_{2,6} + 0.02u_{1,0}$
$= 0.49(0) + 0.49(1.08) + 0.02(0.27)$
$= 0.5346$

$T_{2,1} = 0.49u_{1,0} + 0.49u_{3,0} + 0.02u_{2,0}$
$= 0.49(0.27) + 0.49(2.48) + 0.02(1.08)$
$= 1.3446$

$T_{3,1} = 0.49u_{2,0} + 0.49(u_{4,0} + 0.02u_{3,0}$
$= 0.49(1.08) + 0.49(4.32) + 0.02(2.43)$
$= 2.6946$

$T_{4,1} = 0.49u_{3,0} + 0.49u_{5,0} + 0.02u_{4,0}$
$= 0.49(2.43) + 0.49(6.75) + 0.02(4.32)$
$= 4.5846$

$T_{5,1} = 0.49u_{4,0} + 0.49u_{5,0} + 0.02u_{4,0}$
$= 0.49(4.32) + 0.49(9.72) + 0.02(6.75)$
$= 7.0146$

$T_{6,1} = 0.49u_{5,0} + 0.49u_{6,0} + 0.02u_{5,0}$
$= 0.49[6.75) + 0.49(13.23] + 0.02(9.92)$
$=$

$T_{7,1} = 0.49T_{6,0} + 0.49T_{8,0} + 0.02T_{7,0}$
$= 0.49(9.72) + 0.49(14.25) + 0.02(13.2)$
$= 13.4946$

$T_{8,1} = 0.49T_{7,0} + 0.49T_{9,0} + 0.02T_{8,0}$
$= 0.49(13.28) + 0.49(21.89) + 0.02(19.28)$
$= 19.5446$

$T_{9,1} = 0.49T_{8,0} + 0.49T_{10,0} + 0.02T_{9,0}$
$= 0.49[19.28) + 0.49(22] + 0.02(21.87)$
$= 22.1346$

$T_{10,1} = 0.49T_{9,0} + 0.49T_{11,0} + 0.02T_{10,0}$
$= 0.49(21.87) + 0.49(32.67) + 0.02(27)$
$= 29.2646$

$T_{11,1} = 0.49T_{10,0} + 0.49T_{12,0} + 0.02T_{11,0}$
$= 0.49(27) + 0.49(38.88) + 0.02(32.67)$
$= 32.9346$

$T_{12,1} = 0.49T_{11,0} + 0.49T_{13,0} + 0.02T_{12,0}$
$= 0.49(32.67) = 0.49(45.68) + 0.02(38.88)$
$= 39.1446$

$T_{13,1} = 0.49T_{12,0} + 0.49T_{14,0} + 0.02T_{13,0}$
$= 0.49(38.88) + 0.49(39.2) + 0.02(45.68)$
$= 25.8946$

$T_{14,1} = 0.49T_{13,0} + 0.49T_{15,0} + 0.02T_{14,0}$
$= 0.4(45.63) + 0.49(60.75) + 0.02(52.92)$
$= 53.1846$

$T_{15,1} = 0.49T_{14,0} + 0.49T_{16,0} + 0.02T_{15,0}$
$= 0.49(52.92) + 0.49(69.12) + 0.02(60.75)$
$= 61.0146$

$T_{16,1} = 0.49T_{15,0} + 0.49T_{17,0} + 0.02T_{16,0}$
$= 0.49(60.75) + 0.49(78.05) + 0.02(69.12)$
$= 69.3846$

$T_{17,1} = 0.49T_{16,0} + 0.49T_{18,0} + 0.02T_{19,0}$
$= 0.49(69.12) + 0.49(78.05) + 0.02(69.12)$
$= 69.3846$