

Assignment 5

KUNDE SHARON SAPIEN

PT/ENG103/032

CIVIL ENGINEERING

1) Define mathematical modelling

This is a mathematical representation of a system in order to study its behaviour

2) Methods of obtaining a model

i) Differential equation

ii) use of balance law

(c) From Newton's law of cooling

$$dF/dt = hA(T - T_m)$$

$$dT = -k(T - T_m)dt$$

$$\frac{dT}{T - T_m} = -k dt$$

Integrating both sides

$$\ln(T - T_m) = -kt + C$$

$$T - T_m = e^{-kt+C}$$

$$T - T_m = e^{-kt} \cdot e^C$$

$$\text{let } e^C = C$$

$$T - T_m = e^{kt} \cdot C$$

$$T - T_m = C e^{kt}$$

$$T = C e^{kt} + T_m$$

$$\text{at } t=10, T=10^\circ\text{C} \quad T_m=25^\circ\text{C}$$

$$10 = C e^{k(10)} + 25$$

$$10 = C + 25$$

$$C = 10 - 25$$

$$C = -15$$

$$T = -15 e^{kt} + 25$$

$$\text{at } t=5, T=20^\circ\text{C}$$

$$20 = -15 e^{k(5)} + 25$$

$$20 - 25 = -15 e^{k(5)}$$

$$\frac{-5}{-15} = \frac{-15 e^{k(5)}}{-15}$$

$$\frac{1}{3} = e^{5k}$$

$$\frac{1}{3} = e^{5k}$$

$$5k = \ln(0.3333)$$

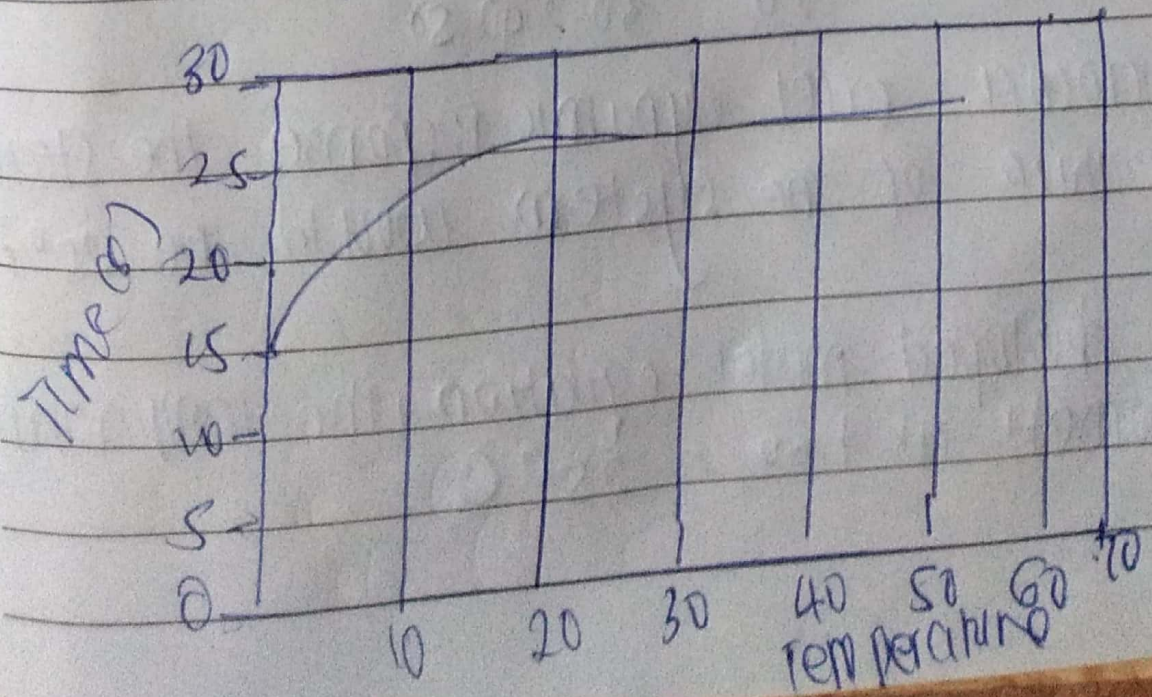
$$\frac{5k}{5} = \frac{\ln(0.3333)}{5}$$

$$k = -0.22$$

$$T(t) = 25 - 15 e^{-0.22t}$$

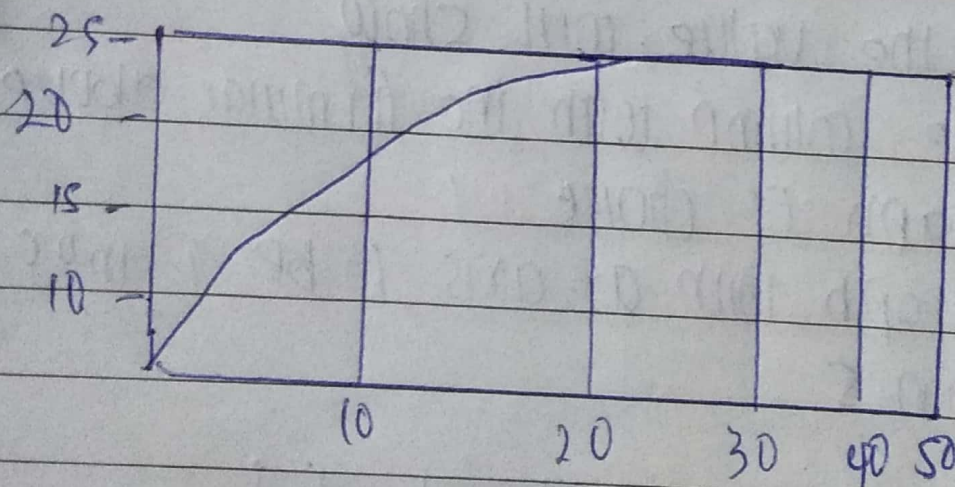
ii) using Microsoft Excel

- pick a box and insert 't'
- pick another box and insert 'r'
- insert a value 0 0 in an empty box in the column 't'
- click on series
- change the series to columns
- insert a stop value of 60
- select the box with 0 and double-click at the edge of the box the values from 0 to 60 appear
- select a box under the column 't'
- insert $= 25 - (15 * \exp(-0.22 * A2))$ into the box and
- click enter, the value will show
- auto fill the column with the formula above
- pick a graph of choice
- label the graph with x-axis to be T in °C and y-axis to be t in s



Using Matlab

- 1) command window
- 2) clear
- 3) clc
- 4) close all
- 5) $t = 0 : 1 : 60$
- 6) $T = 25 - 15 * \exp(-0.22 * t)$
- 7) $T_n = \text{subs}(T)$
- 8) plot (t, T_n)
- 9) grid on
- 10) find minor



(iv) using microsoft excel dynamic response the steady state temperature of the system would be 25°C at 10 s

(v) using the developed model equation, the temperature of the thermometer at $t=0 = 25^{\circ}\text{C}$