

Assignment 7

$$(A) \quad \frac{T_{i,j+1} - T_{i,j}}{\Delta t} = C \times \left[\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{(\Delta x)^2} \right]$$

Where $C = 2.2 \cdot (\text{cm}^2/\text{hr})$

$$T_{i,j+1} - T_{i,j} = \frac{C \Delta t}{(\Delta x)^2} \times [T_{i+1,j} - 2T_{i,j} + T_{i-1,j}]$$

Where $\Gamma = \frac{C \Delta t}{(\Delta x)^2}$

$\rightarrow C = 2.2 \text{ cm}^2/\text{hr}, \Delta t = 0.02 \text{ hr}, \Delta x = 0.3 \text{ cm}$

$$\Gamma = \frac{2.2 \times (0.02)}{(0.3)^2} = 0.48888 \approx 0.49$$

$$T_{i,j+1} - T_{i,j} = 0.49 [T_{i+1,j} - 2T_{i,j} + T_{i-1,j}]$$

$$T_{i,j+1} = 0.49 T_{i+1,j} - 0.98 T_{i,j} + T_{i-1,j} + 0.49 T_{i,j}$$

$$T_{i,j+1} = 0.49 T_{i+1,j} + 0.02 T_{i,j} + 0.49 T_{i-1,j}$$

$$T_{i,j+1} = 0.49 T_{i+1,j} + (-2 + 0.98) T_{i,j} + 0.49 T_{i-1,j}$$

At $t(0) = 0 \text{ hr}$

$$T(x,0) = 3x^2$$

First $T_{0,0}$

Where $x = 0 \text{ cm}$

$$T(0,0) = 3x^2 = 3(0)^2 = 0 \text{ cm} = 0$$

$x = 0.3 \text{ cm}$

$$x_1 \Rightarrow T(0.3, 0) = 3(0.3)^2 = 0.27 \text{ cm}$$

for $x_2 \quad x = 0.6$

$$x_2 \Rightarrow T(0.6, 0) = 3(0.6)^2 = 1.08 \text{ cm}$$

$x = 0.9$

$$x_3 \Rightarrow T(0.9, 0) = 3(0.9)^2 = 2.43 \text{ cm}$$

$x = 1.2 \text{ cm}$

$$x_4 \Rightarrow T(1.2, 0) = 3(1.2)^2 = 4.32 \text{ cm}$$

$x = 1.5 \text{ cm}$

$$x_5 \Rightarrow T(1.5, 0) = 3(1.5)^2 = 6.75 \text{ cm}$$

$x = 1.8 \text{ cm}$

$$x_6 \Rightarrow T(1.8, 0) = 3(1.8)^2 = 9.72 \text{ cm}$$

$$x = 2.1 \text{ cm}$$

$$x_{7.1} \quad T(2.1, 0) = 3(2.1)^2 = 13.23 \text{ cm}$$

$$x = 2.4 \text{ cm}$$

$$x_{8.1} \quad T(2.4, 0) = 3(2.4)^2 = 17.28 \text{ cm}$$

$$x = 2.7 \text{ cm}$$

$$x_{9.1} \quad T(2.7, 0) = 3(2.7)^2 = 21.87 \text{ cm}$$

$$x = 3.0 \text{ cm}$$

$$x_{10.1} \quad T(3.0, 0) = 3(3)^2 = 27 \text{ cm}$$

$$x = 3.3 \text{ cm}$$

$$x_{11.1} \quad T(3.3, 0) = 3(3.3)^2 = 32.67 \text{ cm}$$

$$x = 3.6 \text{ cm}$$

$$x_{12.1} \quad T(3.6, 0) = 3(3.6)^2 = 38.88 \text{ cm}$$

$$x = 3.9 \text{ cm}$$

$$x_{13.1} \quad T(3.9, 0) = 3(3.9)^2 = 45.63 \text{ cm}$$

$$x = 4.2 \text{ cm}$$

$$x_{14.1} \quad T(4.2, 0) = 3(4.2)^2 = 52.92 \text{ cm}$$

$$x = 4.5 \text{ cm}$$

$$x_{15.1} \quad T(4.5, 0) = 3(4.5)^2 = 60.75 \text{ cm}$$

$$x = 4.8 \text{ cm}$$

$$x_{16.1} \quad T(4.8, 0) = 3(4.8)^2 = 69.12 \text{ cm}$$

$$x = 5.1 \text{ cm}$$

$$x_{17.1} \quad T(5.1, 0) = 3(5.1)^2 = 78.03 \text{ cm}$$

$$x = 5.4 \text{ cm}$$

$$T(5.4, 0) = 3(5.4)^2 = 87.48 \text{ cm} \quad x_{18.1}$$

$$x = 5.7 \text{ cm}$$

$$x_{19.1} \quad T(5.7, 0) = 3(5.7)^2 = 97.47 \text{ cm}$$

$$x = 6.0 \text{ cm}$$

$$x_{20.1} \quad T(6.0, 0) = 3(6.0)^2 = 108 \text{ cm}$$

Second row

$$T_{ij+1} = 0.49 T_{i+1,j} + (1 - 2 \times 0.49) T_{i,j} + 0.49 T_{i-1,j}$$

$$\therefore T_{0,j+1} = 0.49 T_{1+1,j} + 0.02 T_{1,j} + 0.49 T_{1-1,j}$$

where $i = 0$ $j = 0$

$$T_{1,2} = 0.49 \times (1.08) + 0.02(0.27) + 0.49(0) = 0.5348$$

$$T_{2,1} = 0.49(2-43) + 0.02(1-08) + 0.47(0-27) = 1.3446$$

where $i=2, j=0$

$$T_{3,1} = 0.47(4-32) + 0.02(2-43) + 0.49(1-98) = 2.6946$$

where $i=3, j=0$

$$T_{4,1} = 0.49(6-75) + 0.02(4-32) + 0.47(2-43) = 4.5846$$

where $i=4, j=0$

$$T_{5,1} = 0.49(9-12) + 0.02(6-75) + 0.49(4-32) = 7.0146$$

where $i=5, j=0$

$$T_{6,1} = 0.49(13-23) + 0.02(9-72) + 0.49(6-75) = 9.9846$$

where $i=6, j=0$

$$T_{7,1} = 0.49(17-28) + 0.02(13-23) + 0.49(9-72) = 13.4946$$

~~$i=7, j=0$~~

$$T_{8,1} = 0.49(21-87) + 0.02(17-28) + 0.49(13-23) = 17.5446$$

~~$i=8, j=0$~~

$$T_{9,1} = 0.49(27) + 0.02(21-87) + 0.49(17-28) = 22.1346$$

$i=9, j=0$

$$T_{10,1} = 0.49(32-67) + 0.02(27) + 0.49(21-87) = 27.2646$$

$i=10, j=0$

$$T_{11,1} = 0.49(38-88) + 0.02(32-67) + 0.49(27) = 32.9346$$

$i=11, j=0$

$$T_{12,1} = 0.49(45-63) + 0.02(38-88) + 0.49(32-67) = 39.1446$$

$i=12, j=0$

$$T_{13,1} = 0.49(52-92) + 0.02(45-63) + 0.49(38-88) = 45.8946$$

$i=13, j=0$

$$T_{14,1} = 0.49(60-75) + 0.02(52-92) + 0.47(45-63) = 53.1846$$

$i=14, j=0$

$$T_{15,1} = 0.49(69-12) + 0.02(60-75) + 0.49(52-92) = 61.0146$$

$i=15, j=0$

$$T_{16,1} = 0.49(78-03) + 0.02(69-12) + 0.47(60-75) = 69.3846$$

$i=16, j=0$

$$i = 17, j = 0$$

$$\begin{aligned} T_{17,1} &= 0.49(87.48) + 0.02(78.03) + 0.49(69.12) \\ &= 78.29 \end{aligned}$$

$$i = 18, j = 0$$

$$T_{18,1} = 0.49(97.47) + 0.02(87.48) + 0.49(78.03) = 87.7466$$

$$i = 19, j = 0$$

$$\begin{aligned} T_{19,1} &= 0.49(108) + 0.02(97.47) + 0.49(87.48) = 97.7366 \\ T_{20,1} &= 108 \end{aligned}$$

-9846

Table of values for $t=0$ for $t=0.02$ s and $d=6$ cm

Time (s)	$x=0$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$x=0$	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0	3.3	3.6	3.9	4.2	4.5	4.8	5.1	5.4	5.7	6.0
$t \rightarrow$	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}
0	0	0.27	1.08	2.43	4.32	6.75	9.72	13.23	17.28	21.87	27.0	32.67	38.88	45.63	52.92	60.75	69.12	78.03	87.48	97.47	108.0
0.02	0	0.534	1.914	4.242	7.518	11.742	16.914	23.034	29.106	35.226	41.394	47.608	53.868	60.174	66.526	72.924	79.368	85.858	92.394	98.976	105.606