

Eng 282 Assignment 5.

(a) What is mathematical modelling

Mathematical modelling can be defined as a description of a system using mathematical concepts and languages. Mathematical models are used in the natural sciences and engineering disciplines as well as the social and physical sciences.

(b) Methods of obtaining a model.

→ Differentiating

→ Use of Balance law

(c) Solution

$$T(0) = 10^\circ\text{C}$$

$$T(\infty) = 20^\circ\text{C}$$

$$\text{Actual temp} = 25^\circ\text{C} = T_A$$

$$\frac{dT}{dt} = k(T - T_A)$$

dt

$$dT = k(T - T_A) dt$$

$$\frac{dT}{T - T_A} = k dt$$

$T - T_A$

Integrating both sides

$$\ln(T - T_A) = kt + C$$

$$T - T_A = e^{kt} + e^C$$

$$1e + e^C = 20 = A$$

$$T - T_A = e^{kt} A$$

$$T - T_A = Ae^{kt}$$

$$T = Ae^{kt} + T_A$$

When $T = 10$

$$10 = Ae^{k(0)} + 25$$

$$10 = A + 25$$

$$A = 10 - 25$$

$$A = -15$$

$$T = 25 - 15e^{5t}$$

$$\text{At } t = 15: \text{ end } T = 20$$

$$20 - 25 = -15e^{5 \cdot 15}$$

$$20 - 25 = -15e^{5 \cdot 15}$$

$$15e^{5 \cdot 15} - 5 = -5e^{5 \cdot 15}$$

$$15e^{5 \cdot 15} = 5$$

$$e^{5 \cdot 15} = 5/15$$

$$e^{5 \cdot 15} = 0.3333$$

$$e^{5 \cdot 15} = \ln 0.3333$$

$$5 \cdot 15 = -1.0987$$

$$k = \frac{-1.0987}{5}$$

$$k = -0.22$$

$$T(t) = 25 - 15e^{-0.22t} \rightarrow \text{model equation}$$

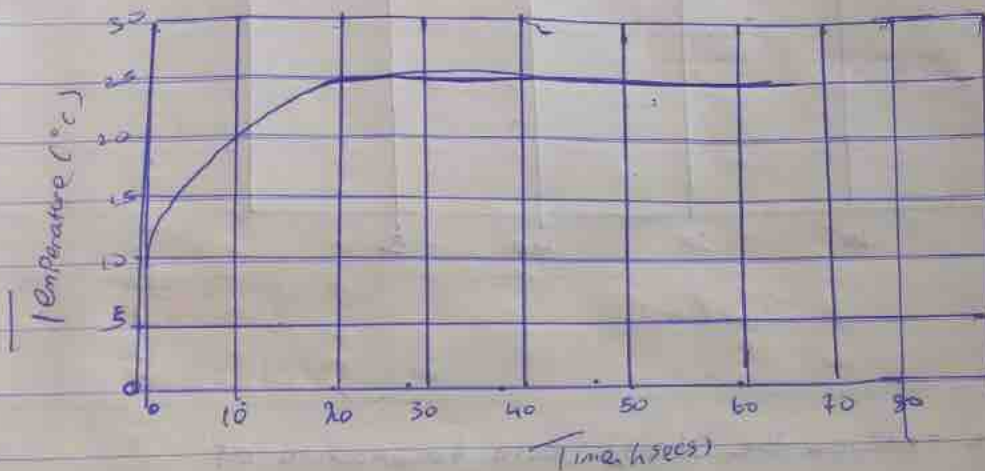
Using Microsoft Excel:

- > Pick a cell insert 't'
- > Pick another cell insert 'T'
- > Under the already labelled cell 't'
- > Insert a value at 0 in an empty cell
- > Click on Series
- > Insert a Step Value of 1
- > Change the Series into columns
- > Insert a Step Value of 60

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- > Under the already labelled $(\text{C} = 2 \text{ } ^\circ \text{T})$
- > Pick a row
- > Inset $\text{C} = 25 (15 \text{ } ^\circ \text{C} (-0.22 \text{ } ^\circ \text{A} 2))$
- > Auto fit
- > Go to inset
- > Pick a graph of choice
- > Label the graph

Output



Using Matlab

Command Window

clear

clc

close all

t = 0:1:60

T = 25 - 15 * exp(-0.22 * t)

plot (t, T)

grid on

grid minor

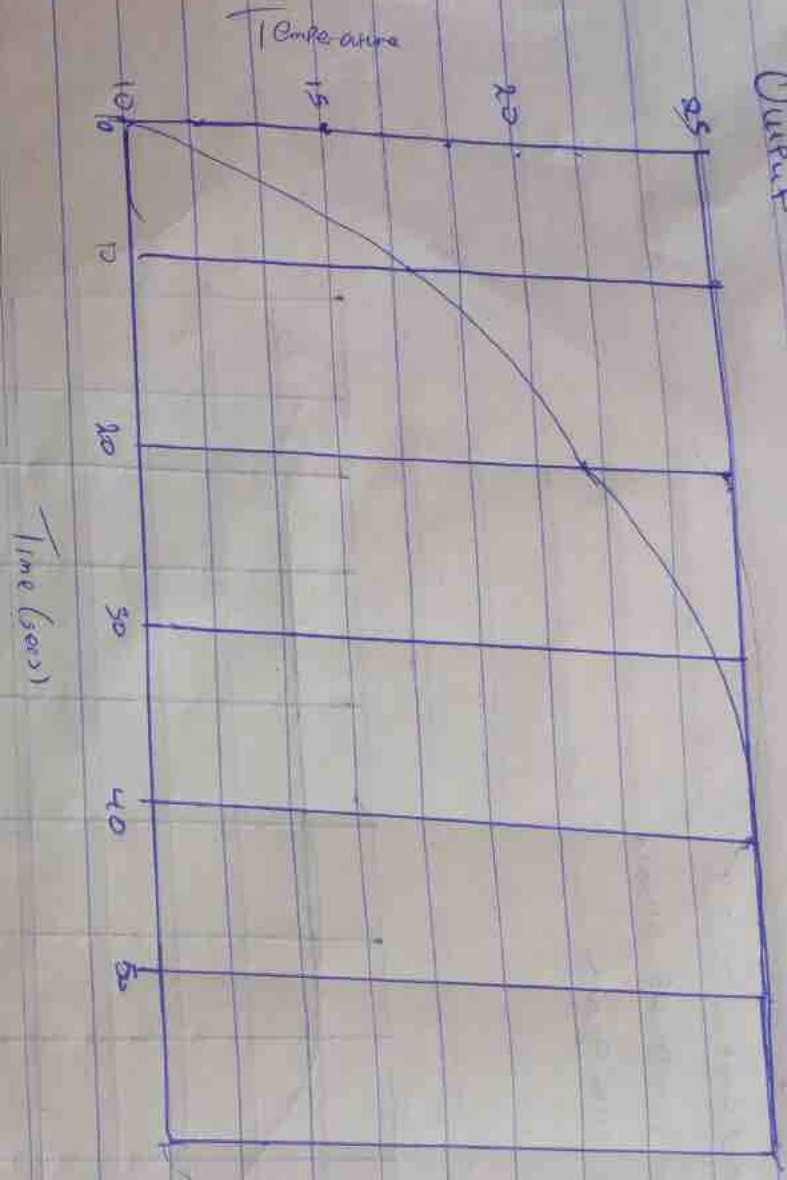
xlabel ('Time (secs)')

ylabel ('Temperature')

grid on

g. & m. m. s.

Disrupt



1) Using first dynamic response the steady state temperature of the system would be 25°C to 10 min.

2) Using developed model eqn, the temperature of the thermocouple at t = 10 min will be 25°C.