

## Assignment 9

The model for the temperature distribution in a rod of length  $L$  is given by eqn

$$\frac{dT(x,t)}{dt} = \frac{d^2 T(x,t)}{dx^2}$$

where  $C_2 = 2 \text{ cm}^2/\text{hr}$

with the conditions and temperature ( $^{\circ}\text{C}$ )

$$T(x,0) = 3x^2$$

$$T(0,t) = 0$$

$$T(L,t) = -108$$

Using  $\Delta t = 0.02 \text{ hr}$  and  $\Delta x = 0.3 \text{ cm}$ , obtain the temperature profile of the system for  $0 \leq t \leq 0.3 \text{ hr}$

manually in tabular form solving up to  $t = 0.02 \text{ hr}$  and  $\Delta x = 6 \text{ cm}$

Soln

Using explicit forward Euler method

$$U_i^{k+1} = \alpha [U_{i+1}^k + U_{i-1}^k] + [1 - 2\alpha] U_i^k$$

where  $i = 1$

$$U_{i,j+1} = \alpha U_{i,j} + \alpha U_{i,j} + (1 - 2\alpha) U_{i,j}$$

$$\text{but } \alpha = \frac{C_2 \Delta t}{(\Delta x)^2} = \frac{2.2 \times 0.02}{0.3^2} = 0.49$$

$$(1 - (2 \times 0.49)) = 0.02$$

for when  $i = 1$  to  $19$

$$U_{1,j+1} = 0.49 U_{0,j} + 0.49 U_{2,j} + 0.02 U_{1,j}$$

$$U_{2,j+1} = 0.49 U_{1,j} + 0.49 U_{3,j} + 0.02 U_{2,j}$$

$$U_{3,j+1} = 0.49 U_{2,j} + 0.49 U_{4,j} + 0.02 U_{3,j}$$

$$U_{4,j+1} = 0.49 U_{3,j} + 0.49 U_{5,j} + 0.02 U_{4,j}$$

$$U_{5,j+1} = 0.49 U_{4,j} + 0.49 U_{6,j} + 0.02 U_{5,j}$$

$$\begin{aligned}
 U_{0,j+1} &= 0.49U_{0j} + 0.49U_{1j} + 0.02U_{6j} \\
 U_{1,j+1} &= 0.49U_{0j} + 0.49U_{2j} + 0.02U_{1j} \\
 U_{2,j+1} &= 0.49U_{1j} + 0.49U_{3j} + 0.02U_{2j} \\
 U_{3,j+1} &= 0.49U_{2j} + 0.49U_{4j} + 0.02U_{3j} \\
 U_{4,j+1} &= 0.49U_{3j} + 0.49U_{5j} + 0.02U_{4j} \\
 U_{5,j+1} &= 0.49U_{4j} + 0.49U_{6j} + 0.02U_{5j} \\
 U_{6,j+1} &= 0.49U_{5j} + 0.49U_{6j} + 0.02U_{6j} \\
 U_{7,j+1} &= 0.49U_{6j} + 0.49U_{7j} + 0.02U_{7j} \\
 U_{8,j+1} &= 0.49U_{7j} + 0.49U_{8j} + 0.02U_{8j} \\
 U_{9,j+1} &= 0.49U_{8j} + 0.49U_{9j} + 0.02U_{9j}
 \end{aligned}$$

for the boundary condition

$$\begin{aligned}
 T(x_0, 0) &= 3x^2 = 3(0.3)^2 = 0.27 & T(x_{14}, 0) &= 3(4.2)^2 = 52.92 \\
 T(x_2, 0) &= 3(0.6)^2 = 1.08 & T(x_{15}, 0) &= 3(4.5)^2 = 60.75 \\
 T(x_3, 0) &= 3(0.9)^2 = 2.43 & T(x_{16}, 0) &= 3(4.8)^2 = 69.12 \\
 T(x_4, 0) &= 3(1.2)^2 = 4.32 & T(x_{17}, 0) &= 3(5.1)^2 = 78.03 \\
 T(x_5, 0) &= 3(1.5)^2 = 6.75 & T(x_{18}, 0) &= 3(5.4)^2 = 87.48 \\
 T(x_6, 0) &= 3(1.8)^2 = 9.72 & T(x_{19}, 0) &= 3(5.7)^2 = 94.74 \\
 T(x_7, 0) &= 3(2.1)^2 = 13.23 & T(x_{20}, 0) &= 3(6)^2 = 108 \\
 T(x_8, 0) &= 3(2.4)^2 = 17.28 \\
 T(x_9, 0) &= 3(2.7)^2 = 21.87 \\
 T(x_{10}, 0) &= 3(3)^2 = 27 \\
 T(x_{11}, 0) &= 3(3.3)^2 = 32.67 \\
 T(x_{12}, 0) &= 3(3.6)^2 = 38.88 \\
 T(x_{13}, 0) &= 3(3.9)^2 = 45.63
 \end{aligned}$$

$$T(0, t) = 0$$

$$T(L, t) = 108$$

temperature has range of 0 to 0.34 with step size of 0.02hr.  $j=0$   
 get to 0.02hr,  $j=0$

When  $T = 0$  [replacing  $v$  with  $T$ ]

$$\begin{aligned}1_{1,1} &= 0.49u_{0,0} + 0.49u_{1,0} + 0.02u_{1,0} \\ &= 0.49(0) + 0.49(1.08) + 0.02(0.29) \\ &= 0.5346\end{aligned}$$

$$\begin{aligned}2_{1,1} &= 0.49u_{1,0} + 0.49u_{2,0} + 0.02u_{2,0} \\ &= 0.49(0.29) + 0.49(2.48) + 0.02(1.08) \\ &= 1.346\end{aligned}$$

$$\begin{aligned}3_{1,1} &= 0.49u_{2,0} + 0.49u_{3,0} + 0.02u_{3,0} \\ &= 0.49(1.08) + 0.49(4.32) + 0.02(2.48) \\ &= 2.6946\end{aligned}$$

$$\begin{aligned}4_{1,1} &= 0.49u_{3,0} + 0.49u_{4,0} + 0.02u_{4,0} \\ &= 0.49(2.48) + 0.49(6.75) + 0.02(4.32) \\ &= 4.5846\end{aligned}$$

$$\begin{aligned}5_{1,1} &= 0.49u_{4,0} + 0.49u_{5,0} + 0.02u_{4,0} \\ &= 0.49(4.32) + 0.49(9.92) + 0.02(6.75) \\ &= 7.0146\end{aligned}$$

$$\begin{aligned}6_{1,1} &= 0.49u_{5,0} + 0.49u_{6,0} + 0.02u_{5,0} \\ &= 0.49(6.75) + 0.49(13.23) + 0.02(9.92) \\ &= \end{aligned}$$

$$\begin{aligned}7_{1,1} &= 0.49T_{6,0} + 0.49T_{7,0} + 0.02T_{7,0} \\ &= 0.49(9.92) + 0.49(17.25) + 0.02(13.2) \\ &= 13.4946\end{aligned}$$

$$\begin{aligned}8_{1,1} &= 0.49T_{7,0} + 0.49T_{8,0} + 0.02T_{8,0} \\ &= 0.49(13.28) + 0.49(21.89) + 0.02(17.28) \\ &= 17.546\end{aligned}$$

$$\begin{aligned}
 T_{9,1} &= 0.49T_{8,0} + 0.49T_{10,0} + 0.02T_{9,0} \\
 &= 0.49(17.28) + 0.49(22) + 0.02(21.87) \\
 &= 22.1346
 \end{aligned}$$

$$\begin{aligned}
 T_{10,1} &= 0.49T_{9,0} + 0.49T_{11,0} + 0.02T_{10,0} \\
 &= 0.49(21.87) + 0.49(32.67) + 0.02(27) \\
 &= 29.2646
 \end{aligned}$$

$$\begin{aligned}
 T_{11,1} &= 0.49T_{10,0} + 0.49T_{12,0} + 0.02T_{11,0} \\
 &= 0.49(27) + 0.49(38.88) + 0.02(32.67) \\
 &= 32.9346
 \end{aligned}$$

$$\begin{aligned}
 T_{12,1} &= 0.49T_{11,0} + 0.49T_{13,0} + 0.02T_{12,0} \\
 &= 0.49(32.67) + 0.49(45.68) + 0.02(38.88) \\
 &= 39.1446
 \end{aligned}$$

$$\begin{aligned}
 T_{13,1} &= 0.49T_{12,0} + 0.49T_{14,0} + 0.02T_{13,0} \\
 &= 0.49(38.88) + 0.49(39.2) + 0.02(45.68) \\
 &= 45.8946
 \end{aligned}$$

$$\begin{aligned}
 T_{14,1} &= 0.49T_{13,0} + 0.49T_{15,0} + 0.02T_{14,0} \\
 &= 0.49(45.68) + 0.49(60.75) + 0.02(52.92) \\
 &= 53.1846
 \end{aligned}$$

$$\begin{aligned}
 T_{15,1} &= 0.49T_{14,0} + 0.49T_{16,0} + 0.02T_{15,0} \\
 &= 0.49(52.92) + 0.49(69.12) + 0.02(60.75) \\
 &= 61.0146
 \end{aligned}$$

$$\begin{aligned}
 T_{16,1} &= 0.49T_{15,0} + 0.49T_{17,0} + 0.02T_{16,0} \\
 &= 0.49(60.75) + 0.49(78.03) + 0.02(69.12) \\
 &= 69.3846
 \end{aligned}$$

$$\begin{aligned}
 T_{17,1} &= 0.49T_{16,0} + 0.49T_{18,0} + 0.02T_{17,0} \\
 &= 0.49(69.12) + 0.49(78.03) + 0.02(69.12) \\
 &= 69.3846
 \end{aligned}$$