

Ardesipc  
Civil Engineering  
Mathematics  
C. 10 250

Q Define mathematical modelling  
Mathematical modelling is a ~~mathematical~~ mathematical representation and simulation of a system which involves solving the model and obtaining its output variable for different values of its input variable or as input variable is changed from one value to another.

Methods of obtaining a model

- Differentiating
- Use of Balance law

Ex (1)

$$T(0) = 10^\circ\text{C}$$

$$T(5) = 20^\circ\text{C}$$

$$\text{Actual temp} = 25^\circ\text{C} = T_a$$

$$\frac{dT}{dt} = k(T - T_a)$$

$$dT = k(T - T_a) dt$$

$$dT \leq k(T - T_a) dt$$

$$\frac{dT}{(T - T_a)} = k dt$$

(Integrating both sides)

$$\ln(T - T_a) = kt + c$$

$$T - T_a = e^{kt} \cdot e^c$$

$$(e^c \cdot e^c) \text{ so } A$$

$$T - T_a = e^{kt} \cdot A$$

$$T - T_a = A e^{kt}$$

$$T = A e^{kt} + T_a$$

$$\text{At } t = 0, T = 10$$

$$10 = A e^{k(0)} + 25$$

$$10 = A + 25$$

$$A = 10 - 25$$

$$A = -15$$

$$T = 25 - 15e^{kt}$$

$$At \quad T(s) = 20$$

$$20 = 25 - 15e^{k(s)}$$

$$20 = 25 - 15e^{5k}$$

$$15e^{5k} = 25 - 20$$

$$15e^{5k} = 5$$

$$e^{5k} = 0.3333$$

$$5k = (\ln 0.3333)$$

$$5k = -1.0986$$

$$k = -0.22$$

$$T(t) = 25 - 15e^{-0.22t}$$

Using Microsoft Excel

Pick a box insert 't'

Pick another box insert 'T'

Under the already labelled box 1 't'

Insert a value at 0 in an empty box

Go to sum fill

Click on series

Insert a step value of 1

Change the series to columns

Insert a step value of 60

Under the already labelled box 2 'T'

Pick a box

Insert " $=25 - (15 * \text{EXP}(-0.22 * A2))$ "

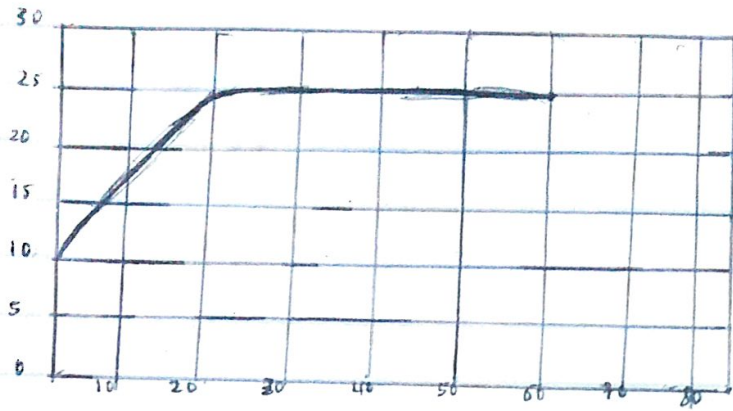
to fill

Go to insert

Pick a graph of choice

Label the graph

Output



Using Matlab

Command Window

clear

clc

close all

t = 0:1:60

T = 25 - 15 \* exp(-0.22 \* t)

plot (t, T)

grid on

grid minor

x label ('Time (sec)')

y label ('Temperature')

grid on

grid minor

iv) Using excel's dynamic response, the steady state temperature of the system would be 25°C at 20 minutes

v) Using the developed model equation, the temperature of the thermometer at  $t \rightarrow \infty$  will be at 25°C

Output

