

GUNOR-NUKI WARE-ERI HARRY

17/ENG06/038

MECHANICAL ENGINEERING

ENG 282

a. Mathematical modelling is a mathematical representation of a system and simulation of a system which involves solving the model and obtaining its output variable for different values of its input variable or as input variable is changed from one value to another.

bi Differentiating

i: Use of balance law

c. Solution

$$T(0) = 10^{\circ}\text{C}$$

$$T(5) = 20^{\circ}\text{C}$$

$$\text{Actual temp} = 25^{\circ}\text{C} = T_a$$

$$\frac{dT}{dt} = k(T - T_a)$$

$$dT = k(T - T_a)dt$$

$$\frac{dT}{(T - T_a)} = kdt$$

Integrating both sides

$$\ln(T - T_a) = kt + C$$

$$T - T_a = e^{kt} + e^C$$

$$\text{let } e^C \text{ be } A$$

$$T - T_a = e^{kt} \cdot A$$

$$T - T_a = Ae^{kt}$$

$$T = Ae^{kt} + T_a$$

When $T = 10$

$$10 = Ae^{k(0)} + 25$$

$$10 = A + 25 ; A = 10 - 25 ; A = -15$$

$$T = 25 - 15e^{kt}$$

$$At\ t(5) = 20$$

$$20 = 25 - 15e^{k(5)}$$

$$20 = 25 - 15e^{5k}$$

$$15e^{5k} = 25 - 20$$

$$15e^{5k} = 5$$

$$e^{5k} = 0.3333$$

$$5k = \ln 0.3333$$

$$5k = -1.0986$$

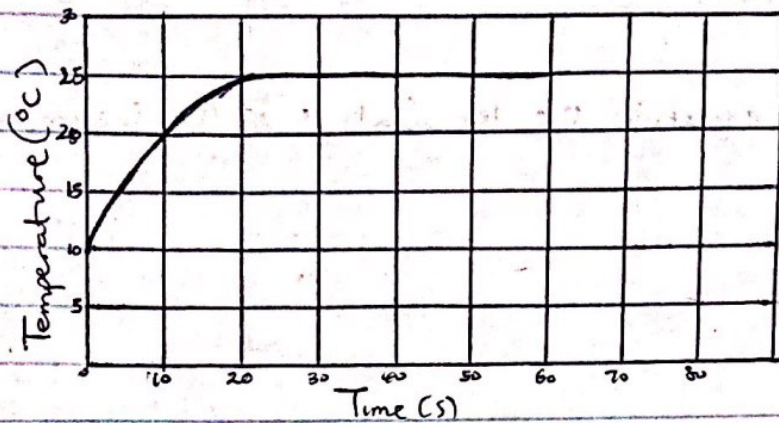
$$k = -0.22$$

$$T(t) = 25 - 15e^{-0.22t}$$

ii. Using Microsoft Excel

- Pick a box, insert 't'
- Pick another box, insert 'T'
- Under the already labelled box 't'
- insert a value of 0 in an empty box
- Go to fill
- Click on Series
- insert a step value of 1
- change the series in to columns
- insert a step value of 60
- Under the already labelled box 2 'T'
- pick a box
- insert $= 25 - (15 * \text{Exp}[0.22 * A2])$
- Auto fill
- Go to insert
- Pick a graph of choice
- Label the graph

Output



Using Matlab

Command Window

Clear

Clc

Close all

t = 0:1:60

T = 25 - 15 * exp(-0.22 * t)

Plot (t, T)

grid on

grid minor

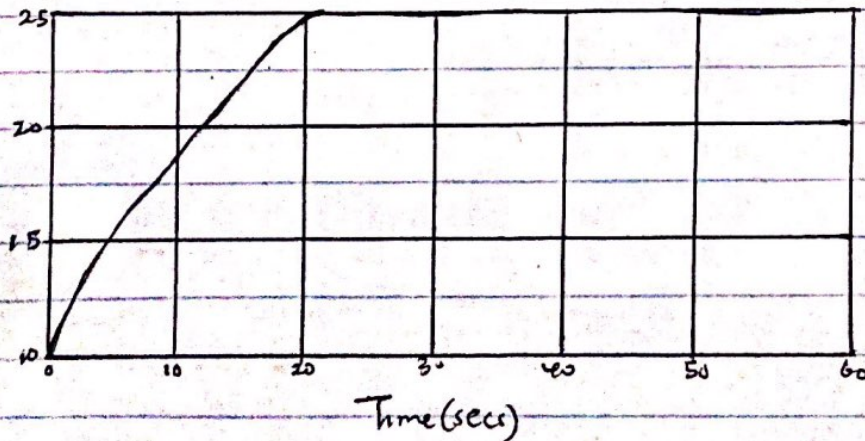
X label ('Time (secs)')

Y label ('Temperature')

grid on

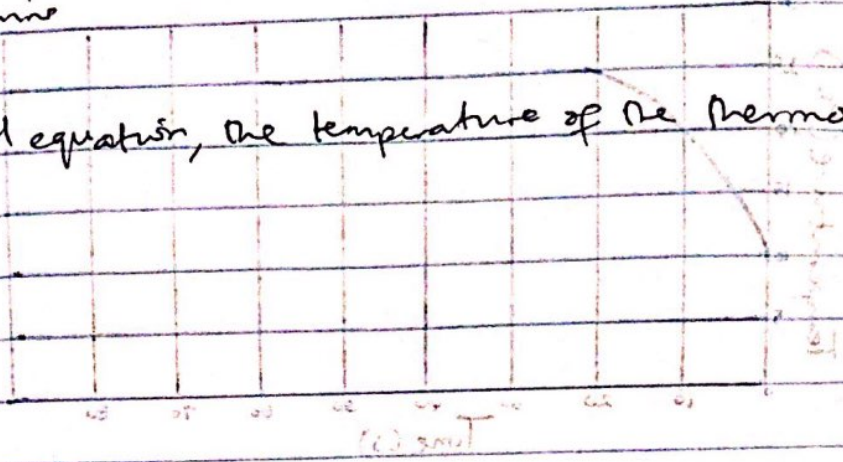
grid minor.

Output.



iv) Using excel's dynamic response, the steady-state temperature of the system would be 25°C in 20 min

v) Using the developed model equation, the temperature of the thermometer at $t = \infty$ will be 25°C .



$T_{\infty} = 25^{\circ}\text{C}$
 $T(t) = T_{\infty} (1 - e^{-t/\tau})$
 $25 = 25 (1 - e^{-20/\tau})$
 $1 = 1 - e^{-20/\tau}$
 $0 = -e^{-20/\tau}$
 $e^{-20/\tau} = 0$
 $-\frac{20}{\tau} = \ln(0)$
 $\tau = \infty$

Output

