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Computer Engineering

$$\frac{dT}{dt}(x,t) = \frac{d^2T}{dx^2}(x,t) \quad ; C = 22 \text{ cm}^2/\text{hr}$$

$$T(x,0) = 3x^2$$

$$T(0,t) = 0$$

$$T(L,t) = 108$$

$$\tau = \frac{C \Delta t}{(\Delta x)^2} = \frac{22 \times 0.02}{0.3^2} = 0.489$$

$$\Delta t = 0.02 \text{ hr} \quad \Delta x = 0.3 \text{ cm}$$

t	x	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0
0.00	0	0	0.27	1.08	2.43	4.32	6.75	9.72	13.20	17.25	21.87	27.00
0.02	0	0.548	1.345	2.695	4.585	7.015	9.98	13.495	17.545	22.135	27.265	

$$\text{at } t=0, x=3x^2 \quad \text{at } t=0, x=0$$

$$\text{at } x=0.3$$

$$x = 3(0.3)^2 = 0.27$$

$$\text{at } x=0.6$$

$$x = 3(0.6)^2 = 1.08$$

$$\text{at } x=0.9$$

$$x = 3(0.9)^2 = 2.43$$

$$\text{at } x=1.2$$

$$x = 3(1.2)^2 = 4.32$$

$$\text{at } x=1.5$$

$$x = 3(1.5)^2 = 6.75$$

$$\text{at } x=1.8$$

$$x = 3(1.8)^2 = 9.72$$

$$\text{at } x=2.1$$

$$x = 3(2.1)^2 = 13.20$$

$$\text{at } x=2.4$$

$$x = 3(2.4)^2 = 17.28$$

$$\text{at } x = 2.2$$

$$x = 3(2.2)^2 = 21.84$$

$$\text{at } x = 3.0$$

$$x = 3(3.0)^2 = 27.00$$

$$\text{at } x = 3.3$$

$$x = (3.3)^2 3 = 32.67$$

$$\text{at } x = 3.6$$

$$x = 3(3.6)^2 = 38.88$$

$$\text{at } x = 3.9$$

$$x = 2(3.9)^2 = 45.63$$

$$\text{at } x = 4.2$$

$$x = 3(4.2)^2 = 52.92$$

$$\text{at } x = 4.5$$

$$x = 3(4.5)^2 = 60.75$$

$$\text{at } x = 4.8$$

$$x = 3(4.8)^2 = 69.12$$

$$\text{at } x = 5.4$$

$$x = 3(5.4)^2 = 87.48$$

$$\text{at } x = 5.7$$

$$x = 3(5.7)^2 = 97.47$$

$$\text{at } x = 6.0$$

$$x = 3(6.0)^2 = 108.00$$

$$\text{at } x = 6.3$$

$$x = 3(6.3)^2 = 119.07$$

$$U_{i,j+1} = U_{i,j} + \alpha U_{i-1,j} + \alpha U_{i+1,j}$$

$$\text{at } T_{1,1}$$

$$T_{1,1} = 0.489 T_{2,0} + 0.489 T_{0,0} + 0.02 T_{1,0}$$

$$T_{1,1} = 0.489 (1.08) + 0.489 (0) + 0.02 (0.27)$$

$$= 0.52812 + 5.4 \times 10^{-3} = 0.5436$$

$$T_{2,1} = 0.489 T_{3,0} + 0.489 T_{1,0} + 0.02 T_{2,0}$$

$$= 0.489 (2.43) + 0.489 (0.27) + 0.02 (1.08)$$

$$1.2203 + 0.0216 = 1.2419$$

$$T_{3,1} = 0.489T_{4,0} + 0.489T_{5,0} + 0.02T_{6,0}$$

$$= 0.489(4.32) + 0.489(1.08) + 0.02(2.43)$$

$$= 2.6406 + 0.5286 = 3.1692$$

$$T_{4,1} = 0.489T_{5,0} + 0.489T_{6,0} + 0.02T_{7,0}$$

$$= 0.489(6.75) + 0.489(2.43) + 0.02(4.32)$$

$$= 3.29925 + 1.18818 + 0.0864 = 4.57383$$

$$T_{5,1} = 0.489T_{6,0} + 0.489T_{7,0} + 0.02T_{8,0}$$

$$= 0.489(9.72) + 0.489(4.32) + 0.02(6.75)$$

$$= 4.75188 + 2.11344 + 0.135 = 6.99032$$

$$T_{6,1} = 0.489T_{7,0} + 0.489T_{8,0} + 0.02T_{9,0}$$

$$= 0.489(13.20) + 0.489(6.75) + 0.02(9.72)$$

$$= 6.4572 + 3.29925 + 0.1944 = 9.95085$$

$$T_{7,1} = 0.489T_{8,0} + 0.489T_{9,0} + 0.02T_{10,0}$$

$$= 0.489(17.22) + 0.489(9.72) + 0.02(13.20)$$

$$= 8.41938 + 4.75188 + 0.264 = 13.43526$$

$$T_{8,1} = 0.489T_{9,0} + 0.489T_{10,0} + 0.02T_{11,0}$$

$$= 0.489(21.87) + 0.489(13.20) + 0.02(17.22)$$

$$= 10.47423 + 6.4572 + 0.3444 = 17.27583$$

$$T_{9,1} = 0.489T_{10,0} + 0.489T_{11,0} + T_{9,0}$$

$$= 0.489(27) + 0.489(17.22) + 0.02(21.87)$$

$$= 13.2963 + 8.41938 + 0.4374 = 22.15308$$

$$T_{10,1} = 0.489T_{11,0} + 0.489T_{9,0} + T_{10,0} \cdot 0.02$$

$$= 0.489(32.67) + 0.489(21.87) + 0.02(27.0)$$

$$= 15.97463 + 10.68543 + 0.54 = 27.20006$$

$$T_{11,1} = 0.489T_{12,0} + 0.02T_{11,0} + 0.489T_{10,0}$$

$$= 0.489(38.88) + 0.02(32.67) + 0.489(27)$$

$$= 18.71232 + 0.6534 + 13.2963 = 32.66202$$

$$T_{12,1} = 0.489T_{13,0} + 0.02T_{12,0} + 0.489T_{11,0}$$

$$= 0.489(45.62) + 0.02(38.88) + 0.489(32)$$

$$= 21.72818 + 0.7776 + 15.648 = 38.15338$$

$$T_{13,1} = 0.489T_{14,0} + 0.02T_{13,0} + 0.489T_{12,0}$$

$$= 0.489(52.92) + 0.02(45.62) + 0.489(38.88)$$

$$= 25.86888 + 0.9124 + 18.99432 = 45.7756$$

$$T_{14,1} = 0.489T_{15,0} + 0.02T_{14,0} + 0.489T_{13,0}$$

$$= 0.489(60.75) + 0.02(5.92) + 0.489(45.63) = 53.1841$$

$$T_{15,1} = 0.489T_{16,0} + 0.02T_{15,0} + 0.489T_{14,0}$$

$$= 0.489(69.12) + 0.02(60.75) + 0.489(52.92) = 61.0146$$

$$T_{16,1} = 0.489T_{17,0} + 0.02T_{16,0} + 0.489T_{15,0}$$

$$= 0.489(78.02) + 0.02(69.12) + 0.489(60.75) = 69.3846$$

$$T_{17,1} = 0.489T_{18,0} + 0.02T_{17,0} + 0.489T_{16,0}$$

$$= 0.489(87.48) + 0.02(78.02) + 0.489(69.12) = 78.2945$$

$$T_{18,1} = 0.489T_{19,0} + 0.02T_{18,0} + 0.489T_{17,0}$$

$$= 0.489(97.45) + 0.02(87.48) + 0.489(78.02) = 87.446$$

$$T_{19,1} = 0.489T_{20,0} + 0.02T_{19,0} + 0.489T_{18,0}$$

$$= 0.489(108) + 0.02(97.45) + 0.489(87.45) = 97.7346$$

$$T_{20,1} = 0.489T_{21,0} + 0.02T_{20,0} + 0.489T_{19,0}$$

$$= 0.489(119.07) + 0.02(108) + 0.489(97.45) = 108.04$$