

② Define Mathematical modelling

This can be defined as a mathematical representation of a system and simulation of a system, which involves solving model and obtaining it's output variables for different values of it's input variables or as input variable is changed from one value to another.

③ Methods of obtaining a model;

- Differentiation
- Use of balance laws

④ Solution

$$T_{\infty} = 100^{\circ}\text{C}$$

$$T_{\infty} = 20^{\circ}\text{C}$$

$$\text{Actual temp} = 25^{\circ}\text{C} = T_a$$

$$\frac{dT}{dt} = k(T - T_a)$$

$$\frac{dt}{(T - T_a)} = k dt$$

integrate

$$\ln(T - T_a) = kt + c$$

$$T - T_a = e^{kt} + e^c$$

$$T - T_a = e^{kt} + e^c$$

$$\text{Let } e^c \text{ be } A$$

$$T - T_a = e^{kt} A$$

$$T - T_a = A e^{kt}$$

$$T = A e^{kt} + T_a$$

where  $T = 10$

$$10 = A e^{k(0)} + 25$$

$$10 = A + 25$$

$$A = 10 - 25$$

$$A = -15$$

$$\text{As } T = 25 - 15 e^{kt}$$

$$A \text{ at } (5) = 20$$

$$20 = 25 - 15e$$

$$20 = 25 - 15e^{5k}$$

$$15e^{5k} = 25 - 20$$

$$15e^{5k} = 5$$

$$e^{5k} = 6.3333$$

$$5k = \ln 6.3333$$

$$5k = 1.8539$$

$$5k = -1.0986$$

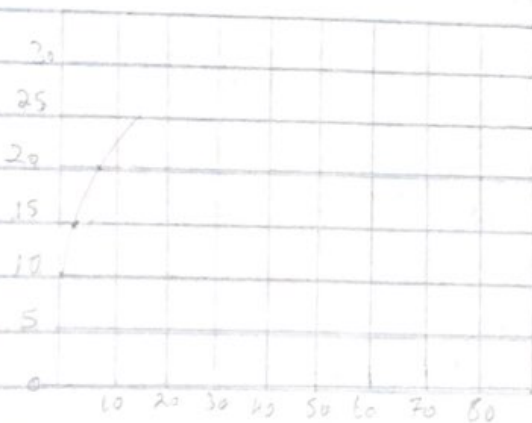
$$k = 0.22$$

$$T(t) = 25 - 15e^{-0.22t} \quad \dots \text{Relating equation}$$

ii) Using Microsoft excel

- Pick a box, insert 't'
- Pick another box, insert 'T'
- Under the already labelled box 't'
- Insert the value of 0 in an empty box
- Select series under the fill tool
- Insert a step value of 1
- Change the series in to columns
- Insert a stop value for 60
- under the already labelled box 'T'
- Insert " $=25 - (15 * \text{Exp}(-0.22 * A2))$ "
- Auto fill
- Go to insert
- Pick a graph of your choice
- Label the graph

Output:



Using MATLAB

- Command window

- Clear

- clc

- Close all

- $t = 0:1:60$

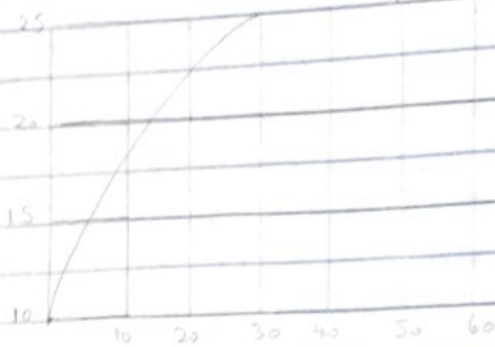
- $T = 25 - 15 * \text{Exp}(-0.22 * t)$

- Plot (t, T)

- Grid on

- Hold on

Output:



iv) Using Excel's dynamic response, the steady state 'temp.' of system would be 25°C at t.

v) Using the developed model equation the 'temp.' of the thermometer at t will be 25°C