

## ASSIGNMENT 7

1. The model for the temperature distribution on a rod of length  $L = 6\text{ cm}$  is as given in equ 1.

$$\frac{dT(x,t)}{dt} = C \frac{d^2 T(x,t)}{dx^2}$$

where  $C = 2.2 \text{ cm}^2/\text{hr}$

with the conditions and the temperature ( $^{\circ}\text{C}$ )

$$T(x, 0) = 3x^2$$

$$T(0, t) = 0$$

$$T(L, t) = -108$$

Using  $\Delta t = 0.02 \text{ hr}$  and  $\Delta x = 0.3 \text{ cm}$ , obtain the temperature profile of the system for  $0 \leq t \leq 0.3 \text{ hr}$

- a) Manually on tabular form solving up to  $t = 0.02 \text{ hr}$  and  $x = 6 \text{ cm}$

### Solution

using Explicit forward Euler method

$$U_i^{k+1} = \gamma [U_{i+1}^k + U_{i-1}^k] + [1 - 2\gamma] U_i^k$$

when  $i = 1$

$$U_{i,j+1} = \gamma U_{2,j} + \gamma U_{0,j} + [1 - 2\gamma] U_{1,j}$$

$$\text{but } \gamma = \frac{C \cdot \Delta t}{\Delta x^2} = \frac{2.2 \times 0.02}{0.3^2} = 0.49$$

$$(1 - (2 \times 0.49)) = 0.02$$

Rewriting the explicit forward euler method

for when  $i = 1$  to  $19$

$$U_{i,j+1} = 0.49 U_{0,j} + 0.49 U_{2,j} + 0.02 U_{1,j}$$

$$u_{2,j+1} = 0.49 u_{1,j} + 0.49 u_{3,j} + 0.02 u_{2,j}$$

$$u_{3,j+1} = 0.49 u_{2,j} + 0.49 u_{4,j} + 0.02 u_{3,j}$$

$$u_{4,j+1} = 0.49 u_{3,j} + 0.49 u_{5,j} + 0.02 u_{4,j}$$

$$u_{5,j+1} = 0.49 u_{4,j} + 0.49 u_{6,j} + 0.02 u_{5,j}$$

$$u_{6,j+1} = 0.49 u_{5,j} + 0.49 u_{7,j} + 0.02 u_{6,j}$$

$$u_{7,j+1} = 0.49 u_{6,j} + 0.49 u_{8,j} + 0.02 u_{7,j}$$

$$u_{8,j+1} = 0.49 u_{7,j} + 0.49 u_{9,j} + 0.02 u_{8,j}$$

$$u_{9,j+1} = 0.49 u_{8,j} + 0.49 u_{10,j} + 0.02 u_{9,j}$$

$$u_{10,j+1} = 0.49 u_{9,j} + 0.49 u_{11,j} + 0.02 u_{10,j}$$

$$u_{11,j+1} = 0.49 u_{10,j} + 0.49 u_{12,j} + 0.02 u_{11,j}$$

$$u_{12,j+1} = 0.49 u_{11,j} + 0.49 u_{13,j} + 0.02 u_{12,j}$$

$$u_{13,j+1} = 0.49 u_{12,j} + 0.49 u_{14,j} + 0.02 u_{13,j}$$

$$u_{14,j+1} = 0.49 u_{13,j} + 0.49 u_{15,j} + 0.02 u_{14,j}$$

$$u_{15,j+1} = 0.49 u_{14,j} + 0.49 u_{16,j} + 0.02 u_{15,j}$$

$$u_{16,j+1} = 0.49 u_{15,j} + 0.49 u_{17,j} + 0.02 u_{16,j}$$

$$u_{17,j+1} = 0.49 u_{16,j} + 0.49 u_{18,j} + 0.02 u_{17,j}$$

$$u_{18,j+1} = 0.49 u_{17,j} + 0.49 u_{19,j} + 0.02 u_{18,j}$$

$$u_{19,j+1} = 0.49 u_{18,j} + 0.49 u_{20,j} + 0.02 u_{19,j}$$

For boundary Condition

$$T(x, 0) = 3x^2 \text{ with } x \text{ ranging from } 0 \text{ to } 6m \text{ with}$$

step size of 0.3

$$T(x_1, 0) = 3x^2 = 3(0.3)^2 = 0.27$$

$$T(x_2, 0) = 3x^2 = 3(0.6)^2 = 1.08$$

$$T(x_3, 0) = 3(0.9)^2 = 2.43$$

$$T(x_4, 0) = 3(1.2)^2 = 4.32$$

$$T(x_5, 0) = 3(1.5)^2 = 6.75$$

$$T(x_6, 0) = 3(1.8)^2 = 9.72$$

$$T(x_7, 0) = 3(2.1)^2 = 13.23$$

$$T(x_8, 0) = 3(2.4)^2 = 17.28$$

$$T(x_9, 0) = 3(2.7)^2 = 21.87$$

$$T(x_{10}, 0) = 3(3.0)^2 = 27.00$$

$$T(x_{11}, 0) = 3(3.3)^2 = 32.67$$

$$T(x_{12}, 0) = 3(3.6)^2 = 38.88$$

$$T(x_{13}, 0) = 3(3.9)^2 = 45.63$$

$$T(x_{14}, 0) = 3(4.2)^2 = 52.92$$

$$T(x_{15}, 0) = 3(4.5)^2 = 60.75$$

$$T(x_{16}, 0) = 3(4.8)^2 = 69.12$$

$$T(x_{17}, 0) = 3(5.1)^2 = 78.03$$

$$T(x_{18}, 0) = 3(5.4)^2 = 87.48$$

$$T(x_{19}, 0) = 3(5.7)^2 = 97.47$$

$$T(x_{20}, 0) = 3(6.0)^2 = 108$$

$$T(0, t) = 0, T(L, t) = 108$$

Temperature has a range of 0 to 0.3 hr with step <sup>size</sup> ~~size~~ 0.02 hr.

To get to 0.2 hr,  $j = 0$

When  $j = 0$  [replacing  $u$  with  $T$ ]

$$\begin{aligned} T_{1,0} &= 0.49 u_{0,0} + 0.49 u_{2,0} + 0.02 u_{1,0} \\ &= 0.49(0) + 0.49(1.08) + 0.02(0.27) \\ &= 0.5349 \end{aligned}$$

$$\begin{aligned} T_{2,0} &= 0.49 u_{1,0} + 0.49 u_{3,0} + 0.02 u_{2,0} \\ &= 0.49(0.27) + 0.49(0.43) + 0.02(1.08) \\ &= 1.3446 \end{aligned}$$

$$T_{3,0} = 2.6946$$

$$T_{11,0} = 32.9346$$

$$T_{4,0} = 4.5846$$

$$T_{12,0} = 39.1446$$

$$T_{5,0} = 7.0146$$

$$T_{13,0} = 45.8946$$

$$T_{6,0} = 9.9846$$

$$T_{14,0} = 53.1846$$

$$T_{7,0} = 13.4946$$

$$T_{15,0} = 61.0146$$

$$T_{8,0} = 17.5446$$

$$T_{16,0} = 69.3846$$

$$T_{9,0} = 22.1346$$

$$T_{17,0} = 78.2946$$

$$T_{10,0} = 27.2646$$

$$T_{18,0} = 87.7446$$

$$T_{19,0} = 97.7346$$

Table for Solving up to  $t = 0.02$  and  $x = 6$  cm

$x$	0	0.3	0.6	0.9	1.2	1.5	1.8
<del><math>T(0)</math></del>	<del>0</del>						
$T(0)$	0	0.27	1.08	2.43	4.32	6.75	9.72
$T(0.02)$	0	0.5346	1.3446	2.6946	4.5846	7.0846	9.9846

$x$	2.1	2.4	2.7	3.0	3.3	3.6	3.9	<del>4.2</del>
<del><math>T(0)</math></del>								
$T(0)$	13.23	17.23	21.87	27.00	32.67	38.88	45.63	52.92
$T(0.02)$	13.4946	17.5446	22.1346	27.2646	32.9346	39.1446	45.8946	53.1846

$x$	4.5	4.8	5.1	5.4	5.7	6.0
<del><math>T(0)</math></del>						
$T(0)$	60.75	69.12	78.03	87.48	97.47	108
$T(0.02)$	61.0146	69.3846	78.2946	87.7446	97.7446	108