The model for the temperature distribution in a rod of length L = 6cm is as given in eqn 1

$$\frac{\partial T(x,t)}{\partial t} = c \frac{\partial^2 T(x,t)}{\partial x^2}$$

where $c = 2.2\ cm^2/hr$

with the conditions and the temperature (°C)

$T(x,0) = 30°$

$T(0,t) = 0$

$T(L,t) = 100°$

using $\Delta t = 0.02hr$ and $\Delta x = 0.3\ cm$, obtain the temperature profile of the system for $0 \le t \le 0.3\ hr$

a) numerically in tabular form solving up to $t = 0.02hr$ and $x = 6\ cm$

Soln

using Explicit forward euler method

$$U_i^{k+1} = r(U_{i+1}^k + U_{i-1}^k) + (1-2r)U_i^k$$

when $i = 1$

$$U_{1,j+1} = r U_{2,j} + r U_{0,j} + (1-2r) U_{1,j}$$

but $r = \frac{c\,\Delta t}{(\Delta x)^2} = \frac{2.2 \times 0.02}{0.3^2}$

$$r = 0.489$$

$$(1 - (2 \times 0.489)) = 0.02$$

Re writing the explicit forward euler method

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for when i = 1 to 19

$U_{1,j+1} = 0.49U_{0,j} + 0.49U_{2,j} + 0.02U_{1,j}$

$U_{2,j+1} = 0.49U_{1,j} + 0.49U_{3,j} + 0.02U_{2,j}$

$U_{3,j+1} = 0.49U_{2,j} + 0.49U_{4,j} + 0.02U_{3,j}$

$U_{4,j+1} = 0.49U_{3,j} + 0.49U_{5,j} + 0.02U_{4,j}$

$U_{5,j+1} = 0.49U_{4,j} + 0.49U_{6,j} + 0.02U_{5,j}$

$U_{6,j+1} = 0.49U_{5,j} + 0.49U_{7,j} + 0.02U_{6,j}$

$U_{7,j+1} = 0.49U_{6,j} + 0.49U_{8,j} + 0.02U_{7,j}$

$U_{8,j+1} = 0.49U_{7,j} + 0.49U_{9,j} + 0.02U_{8,j}$

$U_{9,j+1} = 0.49U_{8,j} + 0.49U_{10,j} + 0.02U_{9,j}$

$U_{10,j+1} = 0.49U_{9,j} + 0.49U_{11,j} + 0.02U_{10,j}$

$U_{11,j+1} = 0.49U_{10,j} + 0.49U_{12,j} + 0.02U_{11,j}$

$U_{12,j+1} = 0.49U_{11,j} + 0.49U_{13,j} + 0.02U_{12,j}$

$U_{13,j+1} = 0.49U_{12,j} + 0.49U_{14,j} + 0.02U_{13,j}$

$U_{14,j+1} = 0.49U_{13,j} + 0.49U_{15,j} + 0.02U_{14,j}$

$U_{15,j+1} = 0.49U_{14,j} + 0.49U_{16,j} + 0.02U_{15,j}$

$U_{16,j+1} = 0.49U_{15,j} + 0.49U_{17,j} + 0.02U_{16,j}$

$U_{17,j+1} = 0.49U_{16,j} + 0.49U_{18,j} + 0.02U_{17,j}$

$U_{18,j+1} = 0.49U_{17,j} + 0.49U_{19,j} + 0.02U_{18,j}$

$U_{19,j+1} = 0.49U_{18,j} + 0.49U_{20,j} + 0.02U_{19,j}$

for the boundary condition

$T(x,0) = 3x^2$ with x running from 0 to 6 (m) with step size of 0.3

$T(x_{1,0}) = 3x^2 = 3\times(0.3)^2 = 0.27$

$T(x_{2,0}) = 3x^2 = 3\times(0.6)^2 = 1.08$

$T(x_{3,0}) = 3x^2 = 3\times(0.9)^2 = 2.43$

$T(x_{4,0}) = 3x^2 = 3\times(1.2)^2 = 4.32$

$T(x_{5,0}) = 3x^2 = 3\times(1.5)^2 = 6.75$

$T(x_{6,0}) = 3x^2 = 3\times(1.8)^2 = 9.72$

$T(x_{7,0}) = 3x^2 = 3\times(2.1)^2 = 13.23$

$T(x_{8,0}) = 3x^2 = 3\times(2.4)^2 = 17.28$

$T(x_{9,0}) = 3x^2 = 3\times(2.7)^2 = 21.87$

$T(x_{10,0}) = 3x^2 = 3\times3^2 = 27$

$T(x_{11,0}) = 3x^2 = 3\times3.3^2 = 32.67$

$T(x_{12,0}) = 3\times3.6^2 = 38.88$

$T(x_{13,0}) = 3\times3.9^2 = 45.63$

$T(x_{14,0}) = 3\times4.2^2 = 52.92$

$T(x_{15,0}) = 3\times4.5^2 = 60.75$

$T(x_{16,0}) = 3\times4.8^2 = 69.12$

$T(x_{17,0}) = 3\times5.1^2 = 78.03$

$T(x_{18,0}) = 3\times5.4^2 = 87.48$

$T(x_{19,0}) = 3\times5.7^2 = 94.7436$

$T(x_{20,0}) = 3\times6^2 = 108$

$T(x_{21,0}) =$

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$T(0,t) = 0$, $T(4,t) = 10\,8$

Temperature has a range of 0 to 0.3 hr with step size of 0.02 hr. To get to 0.02 hr $j = 0$ when $j = 0$ (replacing u with T

$u_{1,2}$.

$$T_{1,1} = 0.49\,u_{0,0} + 0.49\,u_{2,0} + 0.02\,u_{1,0}$$
$$= 0.49(0) + 0.49(1.08) + 0.02(0.27)$$
$$= 0.5346$$

$$T_{2,1} = 0.49\,u_{1,0} + 0.49\,u_{3,0} + 0.02\,u_{2,0}$$
$$= 0.49(0.27) + 0.49(2.43) + 0.02(1.08)$$
$$= 1.3446$$

$$T_{3,1} = 0.49\,u_{2,0} + 0.49\,u_{4,0} + 0.02\,u_{3,0}$$
$$= 0.49 \times (1.08) + 0.49(4.32) + 0.02(2.43)$$
$$= 2.6946$$

$$T_{4,1} = 0.49\,u_{3,0} + 0.49\,u_{5,0} + 0.02\,u_{4,0}$$
$$= 0.49 \times (2.43) + 0.49(6.75) + 0.02(4.32)$$
$$= 4.5846$$

$$T_{5,1} = 0.49\,u_{4,0} + 0.49\,u_{6,0} + 0.02\,u_{5,0}$$
$$= 0.49 \times (4.32) + 0.49(9.72) + 0.02(6.75)$$
$$= 7.0146$$

$$T_{6,1} = 0.49\,T_{5,0} + 0.49\,T_{7,0} + 0.02\,T_{6,0}$$
$$= 0.49 \times (6.75) + 0.49(13.23) + 0.02(9.72)$$
$$= 9.9846$$

$$T_{7,1} = 0.49\,T_{6,0} + 0.49\,T_{8,0} + 0.02\,T_{7,0}$$
$$= 0.49 \times (9.72) + 0.49(17.28) + 0.02(13.23)$$
$$= 13.4946$$

$$T_{8,1} = 0.49\,T_{7,0} + 0.49\,T_{9,0} + 0.02\,T_{8,0}$$
$$= 0.49 \times (13.23) + 0.49(21.87) + 0.02(17.28)$$
$$= 17.5446$$

$$T_{9,1} = 0.49\,T_{8,0} + 0.49\,T_{10,0} + 0.02\,T_{9,0}$$
$$= 0.49(17.28) + 0.49(27) + 0.02(21.87)$$
$$= 22.1346$$

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$T_{10,1} = 0.49\,T_{9,0} + 0.49\,T_{11,0} + 0.02\,T_{10,0}$

$0.49(21.87) + 0.49(32.67) + 0.02(27)$

$= 27.264$

$T_{11,1} = 0.49\,T_{10,0} + 0.49\,T_{12,0} + 0.02\,T_{11,0}$

$0.49(27) + 0.49(38.88) + 0.02(32.67)$

$= 32.934$

$T_{12,1} = 0.49\,T_{11,0} + 0.49\,T_{13,0} + 0.02\,T_{12,0}$

$0.49(32.67) + 0.49(45.63) + 0.02 \times 38.88$

$= 39.144$

$T_{13,1} = 0.49\,T_{12,0} + 0.49\,T_{14,0} + 0.02\,T_{13,0}$

$0.49(38.88) + 0.49(59.2) + 0.02(45.63)$

$= 45.894$

$T_{14,1} = 0.49\,T_{13,0} + 0.49\,T_{15,0} + 0.02\,T_{14,0}$

$0.49(45.63) + 0.49(60.75) + 0.02(52.92)$

$= 53.184$

$T_{15,1} = 0.49\,T_{14,0} + 0.49\,T_{16,0} + 0.02\,T_{15,0}$

$0.49(52.92) + 0.49(69.12) + 0.02(60.75)$

$= 61.014$

$T_{16,1} = 0.49\,T_{15,0} + 0.49\,T_{17,0} + 0.02\,T_{16,0}$

$= 0.49(60.75) + 0.49(78.03) + 0.02(69.12)$

$= 69.384$

$T_{17,1} = 0.49\,T_{16,0} + 0.49\,T_{18,0} + 0.02\,T_{17,0}$

$= 0.49(69.12) + 0.49(87.48) + 0.02(78.03)$

$= 78.294$

$T_{18,1} = 0.49\,T_{17,0} + 0.49\,T_{19,0} + 0.02\,T_{18,0}$

$= 0.49(78.03) + 0.49(97.47) + 0.02(87.48)$

$= 87.744$

$T_{19,1} = 0.49\,T_{18,0} + 0.49\,T_{20,0} + 0.02\,T_{19,0}$

$= 0.49(87.48) + 0.49(108) + 0.02(97.47)$

$= 97.734$

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x	0	0.3	0.6	0.9	1.2	1.5	1.8	2.1	2.4	2.7	3.0	3.3	3.6	3.9	4.2	4.5
T) 0	0	0.27	1.08	2.43	4.32	6.75	9.72	13.23	17.28	21.87	27	32.67	38.88	45.63	52.92	60.75
0.02	0	0.5346	1.3446	2.6946	4.5846	7.0146	9.9846	13.4946	17.5446	22.1346	27.2646	32.9346	39.1446	45.8946	53.1846	61.0146

x	4.8	5.1	5.4	5.7	6.0
T 0	69.12	78.03	87.48	97.47	108
0.02	69.3846	78.3246	87.7446	97.7346	108

Table for solving up to $t = 0.02$hr and $x = 6$ cm

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