

(a) Mathematical modelling is defined as the mathematical representation of a system and simulation of a system involves solving the model and obtaining its mathematical solution and its physical solution.

(b) By differentiation

- By the use of balance law

- By the use of Newton's law of cooling

(c) From Newton's law of cooling

$$\frac{dT}{dt} \propto (T - T_A)$$

$$\frac{dT}{dt} = u(T - T_A)$$

$$\frac{dT}{(T - T_A)} = u dt$$

integrate both sides

$$\ln(T - T_A) = Kt + C$$

$$-(T - T_A) = e^{Kt+C}$$

$$(T - T_A) = e^{Kt} \cdot e^C$$

$$\text{But } e^C = T_0$$

$$T - T_A = T_0 e^{Kt} \quad \text{--- ①}$$

initially at $t=0$, $T=10^\circ\text{C}$ and actual temp. $T_A = 25^\circ\text{C}$

equation ① becomes

$$10 - 25 = T_0 e^{K(0)}$$

$$-15 = T_0 \times e^0 = T_0$$

$$T_0 = -15^\circ\text{C}$$

$$\therefore T = T_A + T_0 e^{Kt}$$

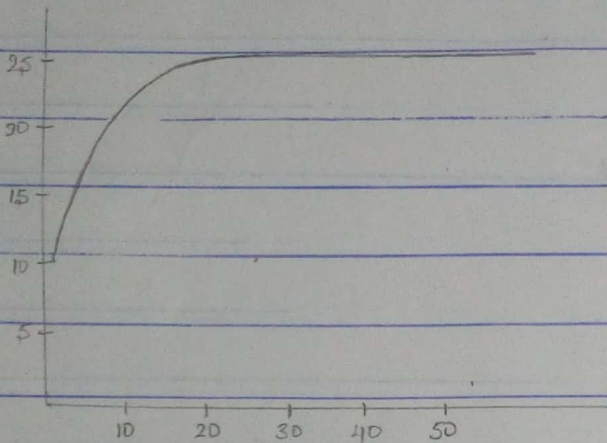
$$-5 = -15e$$

$$\ln(1/3) = 5u$$

$$k = \ln(1/3)/5 = -0.22$$

∴ The model for the system

$$T = 25 - 15e^{-0.22t}$$



iii) command window

```
clear
```

```
clc
```

```
close all
```

```
t = 0:0.5:50
```

```
T = 25 - 15 * exp(-0.22 * t)
```

```
Plot (t, T)
```

```
grid on
```

```
grid minor
```

```
x label ("Time (sec)")
```

```
y label ("Temperature (C)")
```

iv) The steady-state temperature of the system is 25°C .

v₂ At the temperature of 25°C, it was 0.5 second. There was no change in the temperature despite the increase in the time, therefore, the system is said to be stable at low temperature.

v₃ For $t = ?$ and $t = 24.9^\circ\text{C}$

From the equation $T = 25 - 15e^{-0.22t}$

$$24.9 = 25 - 15e^{-0.22t}$$

$$0.1 = 15e^{-0.22t}$$

$$t = 22.76 \text{ mins}$$

$$0.76 \text{ mins} \rightarrow \text{secs}$$

$$0.76 \times 60 = 45.6 \rightarrow 46 \text{ secs}$$

\therefore The time required is 22 mins 46 secs