

1) Mathematical modelling can be defined as the mathematical representation of a system and simulation of a system involves solving the model and obtaining its mathematical solution and its physical solution.

ii) By the use of balance law

iii) By differentiation

iv) The use of Newton law of cooling

2) From Newton's law of cooling states

$$\frac{dT}{dt} \propto (T - T_a)$$

$$\frac{dT}{dt} = -u(T - T_a)$$

$$\frac{dT}{(T - T_a)} = -u dt$$

integrate both sides

$$\ln(T - T_a) = -kt + C$$

$$-(T - T_a) = e^{-kt + C}$$

$$(T - T_a) = e^{-kt} \cdot e^C$$

$$\text{But } e^C = T_0$$

$$T - T_a = T_0 e^{-kt} \quad \text{--- (1)}$$

initially at  $t = 0$   $T = 10^\circ\text{C}$

and actual temperature

$T_a = 25^\circ\text{C}$  equation one becomes

$$10 - 25 = T_0 e^{-k \cdot 0}$$

$$-15 = T_0 \times 1 = T_0$$

$$\therefore T = T_a + T_0 e^{-kt}$$

at  $t = 5 \text{ min}$   $T = 20^\circ\text{C}$

from  $t = 25 - 15 e^{-kt}$

$$20 = 25 - 15 e^{-5k}$$

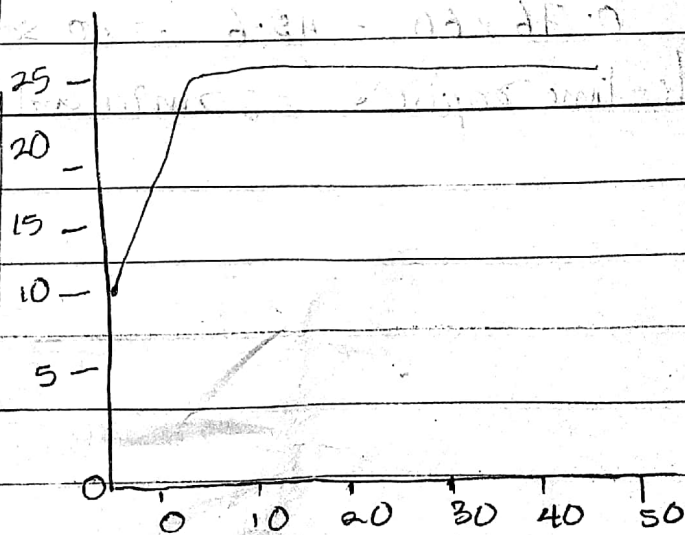
$$-5 = -15 e^{-5k}$$

$$\ln(1/3) = -5k$$

$$k = \ln(1/3) / 5 = -0.22$$

$\therefore$  The model for the system

$$T = 25 - 15 e^{-0.22t}$$



Command window

clear

clc

close all

t = 0:5:50

$$T = 25 - 15e^{-0.22t}$$

plot  $(t, T)$

grid on

grid mirror

x label ("Time (sec)")

y label ("Temperature (°C)")

The steady state temperature of the system = 25°C.

5) At the temp of 25°C it was 0.5 seconds that there was no change in the temp despite the increase in the time therefore the system is said to be stable at low temp.

6) for  $t = ?$        $T = 24.9^\circ\text{C}$

from equation  $T = 25 - 15e^{-0.22t}$

$$24.9 = 25 - 15e^{-0.22t}$$

$$0.1 = 15e^{-0.22t}$$

$$t = 22.76 \text{ mins}$$

0.76 mins  $\rightarrow$  seconds

$$0.76 \times 60 = 45.6 \rightarrow 46 \text{ seconds}$$

The time required = 22 min and 46 sec.