

a) Mathematical modeling is the art of translating problems from an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insight, answers and guidance useful for the originating application.

b) The two methods are:

- Differentiating
- Use of balance Law.

c) (i) $T(0) = 10t$
 $T(1) = 20^\circ\text{C}$
Actual Temperature = $25^\circ\text{C} = T_h$

$$\frac{dT}{dt} = K(T - T_0)$$

$$dT = K(T - T_0)dt$$

$$\frac{dT}{(T - T_A)} = Kt + C$$

$$\lim (T - T_A) = Kt + C$$

$$T - T_A = e^{Kt} + e^C$$

$$\text{Let } e^C = A$$

$$T - T_A = Ae^{Kt}$$

$$T = T_A + Ae^{Kt}$$

$$T = Ae^{Kt} + T_h$$

$$\text{when } t = 10$$

$$10 = Ae^{K(10)} + 25$$

$$10 = A + 25$$

$$A = -15$$

$$T = 25 - 15e^{Kt}$$

$$\text{at } t(5) = 20$$

$$20 = 25 - 15e^{K(5)}$$

$$15 e^{5k} = 25 - 20$$

$$15 e^{5k} = 5$$

$$e^{5k} = 5/15$$

$$= 0.3333$$

$$5k = \ln 0.3333$$

$$5k = -1.0986$$

$$k = -1.0986/5$$

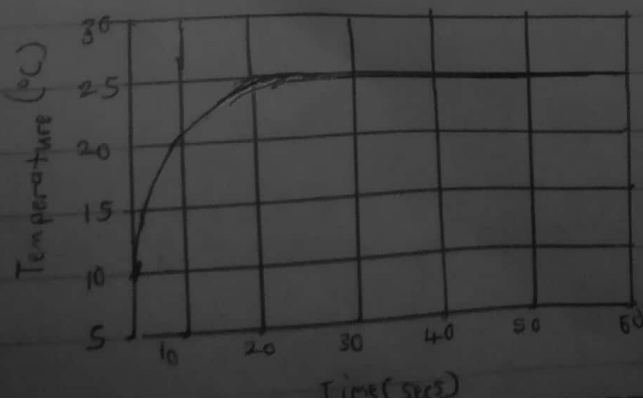
$$= -0.22$$

$$T(t) = 25 - 15e^{-0.22t}$$

i) USING MICROSOFT EXCEL

- Put a box and insert 't'
- Put another box and insert 'T'
- Under the already labelled box 't'
- Insert 0 in an empty box
- Insert a step value of $\frac{1}{10}$
- Arrange the series in ~~the~~ 10 columns.
- Under the labelled box 't'
- Select a box and insert $= 25 - (15 * \text{EXP}(-0.22 * A2))$
- Auto fill
- Go to insert
- Plot a graph of choice (x and y scatter properly)
- Label the graph.

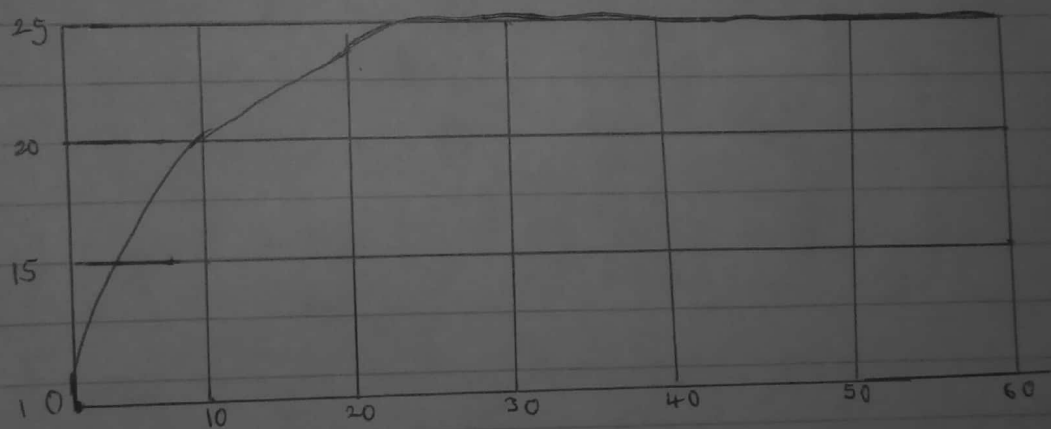
OUTPUT



i) USING MATLAB

- Command window
- clear
- clc
- close all
- $t = 0:1:60$
- $T = 25 - 150 \exp(-0.22 * t)$
- Plot (t, T)
- Grid on
- Grid minor
- X Label ('Time (secs)')
- Y Label ('Temperature')
- Grid on
- Grid minor.

OUTPUT



iv) The steady state temperature of the system would be at 25°C at 20 minutes.

v) Using the developed model equation, the temperature of the thermometer at $t \rightarrow \infty$ will be ~~25~~ 25°C .