

Mathematical Modelling is the art of translating problems from an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insight, answers and guidance with for originating applications.

(b) The two methods are:

- Differentiating method.
- Use of balance law.

(i) $T(0) = 10$

$T(1) = 20^\circ\text{C}$

Actual Temperature = $25^\circ\text{C} = \bar{T}_A$

$$\frac{dT}{dt} = k(T - T_0)$$

$$dT = k(T - T_0) dt$$

$$\frac{dT}{(T - T_A)} = kt + c$$

$(T - T_A)$

$$\lim (T - T_A) = kt + c$$

$$T - T_A = A e^{kt} + e^c$$

let $e^c = A$

$$T - T_A = A e^{kt}$$

$$T = T_A + A e^{kt}$$

$$T = T_A + A e^{kt}$$

$$T = A e^{kt} + T_A$$

when $T = 10$

$$10 = A e^{k(0)} + 25$$

$$10 = A + 25$$

$$A = -15$$

$$T = 25 - 15 e^{kt}$$

at $t(5) = 20$

$$20 = 25 - 15 e^{k(5)}$$

$$15 e^{5k} = 25 - 20$$

$$15 e^{5k} = 5$$

$$e^{5k} = \frac{5}{15}$$

$$e^{5k} = 0.3333$$

$$5k = \ln 0.3333$$

$$5k = -1.0986$$

$$k = \frac{-1.0986}{5}$$

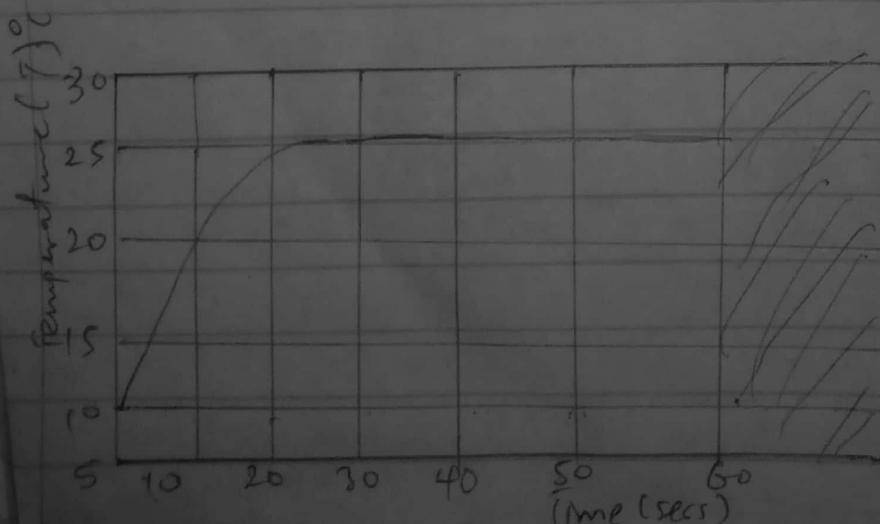
$$k = 0.22$$

$$T(t) = 25 - 15e^{-0.22t}$$

ii) Using Microsoft Excel.

- * put a box and insert "t"
- * put another box and insert "T"
- * under the already labelled box "t"
- * insert "0" in an empty box.
- * insert a step value of 1
- * Arrange the series in 10 columns.
- * Under the labelled box "t"
- * select a box and insert $[25 - (15 * \text{Exp}(-0.22 * A2))]$
- * Auto fill it.
- * Go to insert.
- * plot a graph of choice (sc and y scatter property).
- * label the graph.

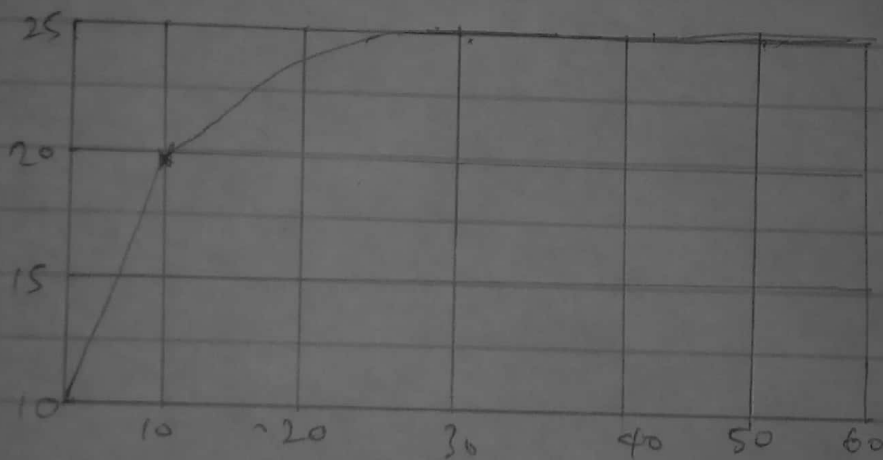
Output



(iii) Using Matlab.

- * Command window
- * clear
- * clc
- * close all
- * t = 0:1:60
- * T = 25 - 150 * exp(-0.22 * t)
- * plot (t, T)
- * Grid On
- * Grid minor
- * X label ('Time (secs)')
- * Y label ('Temperature')
- * Grid On
- * Grid minor

Output



(iv) The steady state temperature of the system would be at 25°C at 20 mins.

(v) Using the developed model equation, the temperature of the thermocouple at $t \rightarrow \infty$ will be 25°C .