Question 1 [20 Marks]

The model for the temperature distribution in a rod of length L = 6m is as given in Equation (1),

$\frac{\partial T(x,t)}{\partial t} = c\frac{\partial^2 T(x,t)}{\partial x^2}$ ……(1)

where

c = 2.2 cm^2 /hr ……(2)

with the conditions that the temperature (℃):

T(x, 0) = 3x^2…… (3)

T(0, t) = 0 ……(4)

T(L,t) = 108 ……(5)

Using Δt= 02 .0 hr and Δx =3.0 cm, obtain the temperature profile of the system for 0 ≤ t ≤ 3 .0 hr

(a) manually, in tabular form, solving up to t =0 .02 hr and x = 6cm

SOLUTION

$\frac{\partial T(x,t)}{\partial t} = c\frac{\partial^2 T(x,t)}{\partial x^2}$

Simplifying the above equation using the forward difference method for first order and the central difference for the second order; gives:

$\frac{T_{i+1,J}-T_{i,j}}{\Delta t} = c\frac{T_{i,j+1}-2T_{i,j}+T_{i,j-1}}{(\Delta x)^2}$

$T_{i+1,j} - T_{i,j} = \frac{c\Delta t}{(\Delta x)^2}\ (T_{i,j+1}-2T_{i,j}+T_{i,j-1})$

Let r = = $\frac{c\Delta t}{(\Delta x)^2}$

$T_{i+1,j} - T_{i,j} = r\ (T_{i,j+1}-2T_{i,j}+T_{i,j-1})$

$T_{i+1,j} = r\ (T_{i,j+1}-2T_{i,j}+T_{i,j-1}) + T_{i,j}$

$T_{i+1,j} = r\ T_{i,j+1}+(1-2r)\ T_{i,j}+rT_{i,j-1}$

The boundary conditions are:

T(0, t) = 0

T(L,t) = 108

And L = 6cm

T(x, 0) = 3x^2

c = 2.2 cm^2 /hr

Δt = 0.02 hr

Δx = 0.3cm

r = $\frac{c\Delta t}{(\Delta x)^2} = \frac{2.2*0.02}{0.3^2}$ = 0.4889

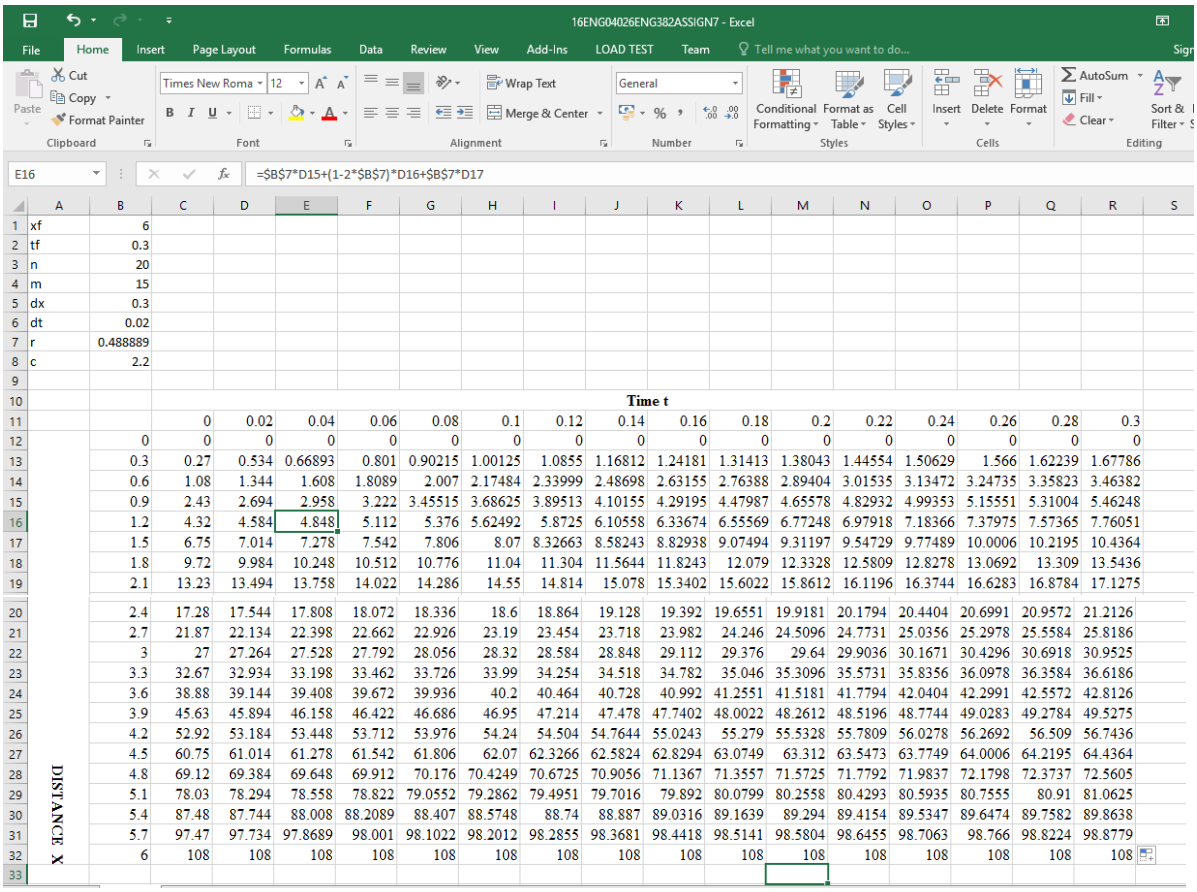
$nx = \frac{x}{\Delta x} = \frac{6}{0.3}$ = 20

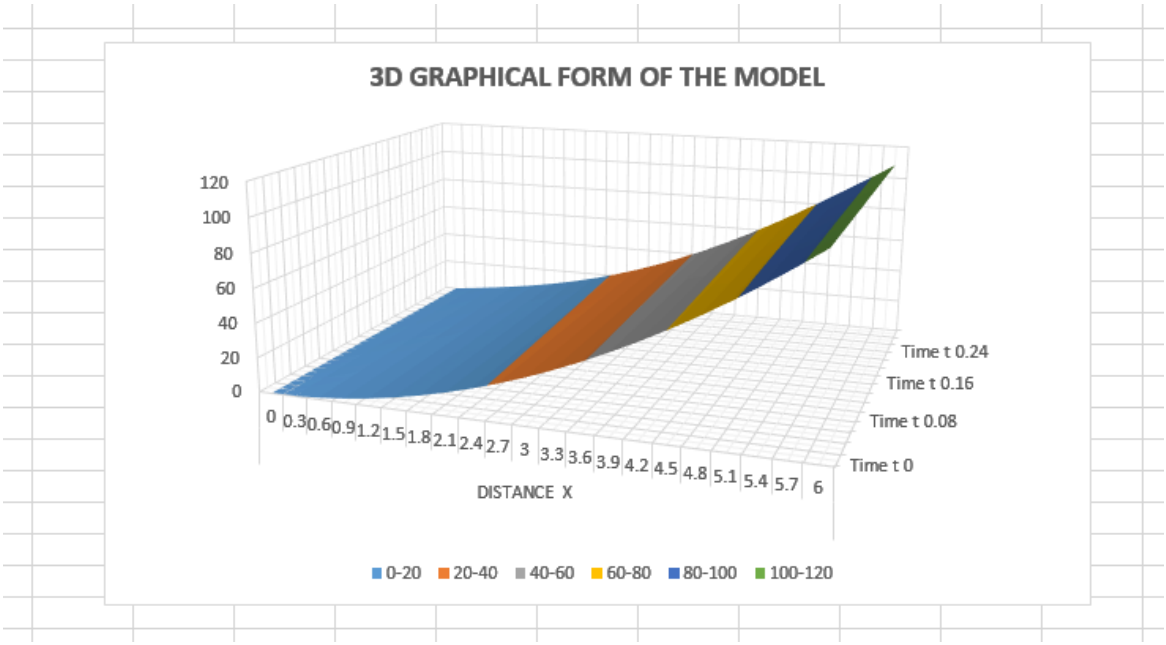
$nt = \frac{t}{\Delta t} = \frac{0.3}{0.02}$ = 15

	TIME t VALUES			
Distance x values		i	0	1
	j		0.00	0.02
	0	0.0	0	0
	1	0.3	0.27	0.534

	2	0.6	1.08	1.344
	3	0.9	2.43	2.694
	4	1.2	4.32	4.584
	5	1.5	6.75	7.014
	6	1.8	9.72	9.984
	7	2.1	13.23	13.494
	8	2.4	17.28	17.544
	9	2.7	21.87	22.134
	10	3.0	27.00	27.264
	11	3.3	32.67	32.934
	12	3.6	38.88	39.144
	13	3.9	45.63	45.894
	14	4.2	52.92	53.184
	15	4.5	60.75	61.014
	16	4.8	69.12	69.384
	17	5.1	78.03	78.294
	18	5.4	87.48	87.744
	19	5.7	97.47	97.734
	20	6.0	108.00	108

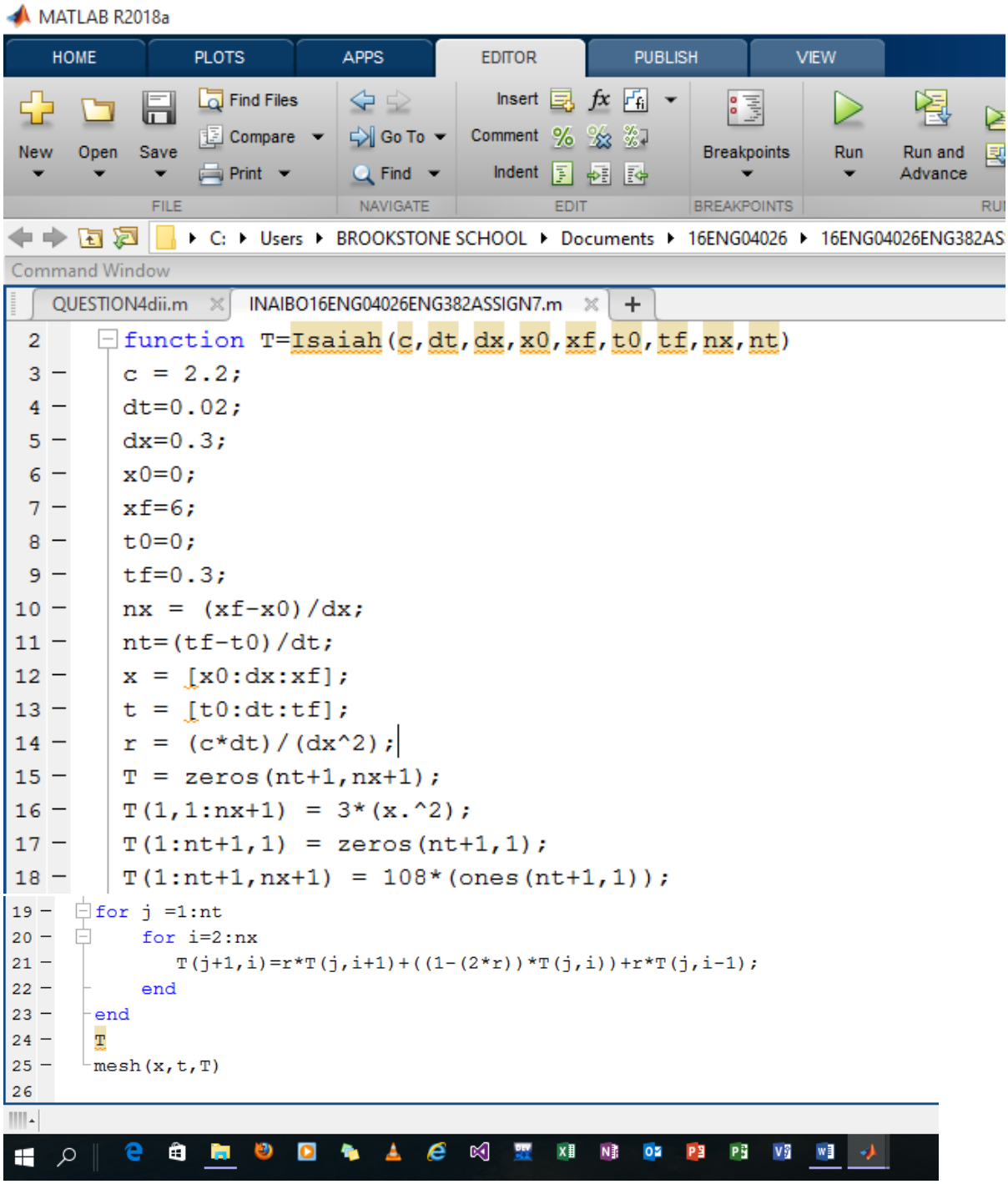
(b) with the aid of Microsoft Excel, in tabular and 3D graphical forms, and SOLUTION





(c) with the aid of MATLAB, in tabular and 3D graphical forms.

SOLUTION


MATLAB R2018a
HOME
PLOTS
APPS
EDITOR
PUBLISH
VIEW
New
Open
Save
Find Files
Compare
Print
Go To
Find
Insert
Comment
Indent
Breakpoints
Run
Run and Advance
FILE
NAVIGATE
EDIT
BREAKPOINTS
C: ▸ Users ▸ BROOKSTONE SCHOOL ▸ Documents ▸ 16ENG04026 ▸ 16ENG04026ENG382AS
Command Window
QUESTION4dii.m
INAIBO16ENG04026ENG382ASSIGN7.m
2 function T=Isaiah(c,dt,dx,x0,xf,t0,tf,nx,nt)
3 c = 2.2;
4 dt=0.02;
5 dx=0.3;
6 x0=0;
7 xf=6;
8 t0=0;
9 tf=0.3;
10 nx = (xf-x0)/dx;
11 nt=(tf-t0)/dt;
12 x = [x0:dx:xf];
13 t = [t0:dt:tf];
14 r = (c*dt)/(dx^2);
15 T = zeros(nt+1,nx+1);
16 T(1,1:nx+1) = 3*(x.^2);
17 T(1:nt+1,1) = zeros(nt+1,1);
18 T(1:nt+1,nx+1) = 108*(ones(nt+1,1));
19 for j =1:nt
20 for i=2:nx
21 T(j+1,i)=r*T(j,i+1)+((1-(2*r))*T(j,i))+r*T(j,i-1);
22 end
23 end
24 T
25 mesh(x,t,T)
26

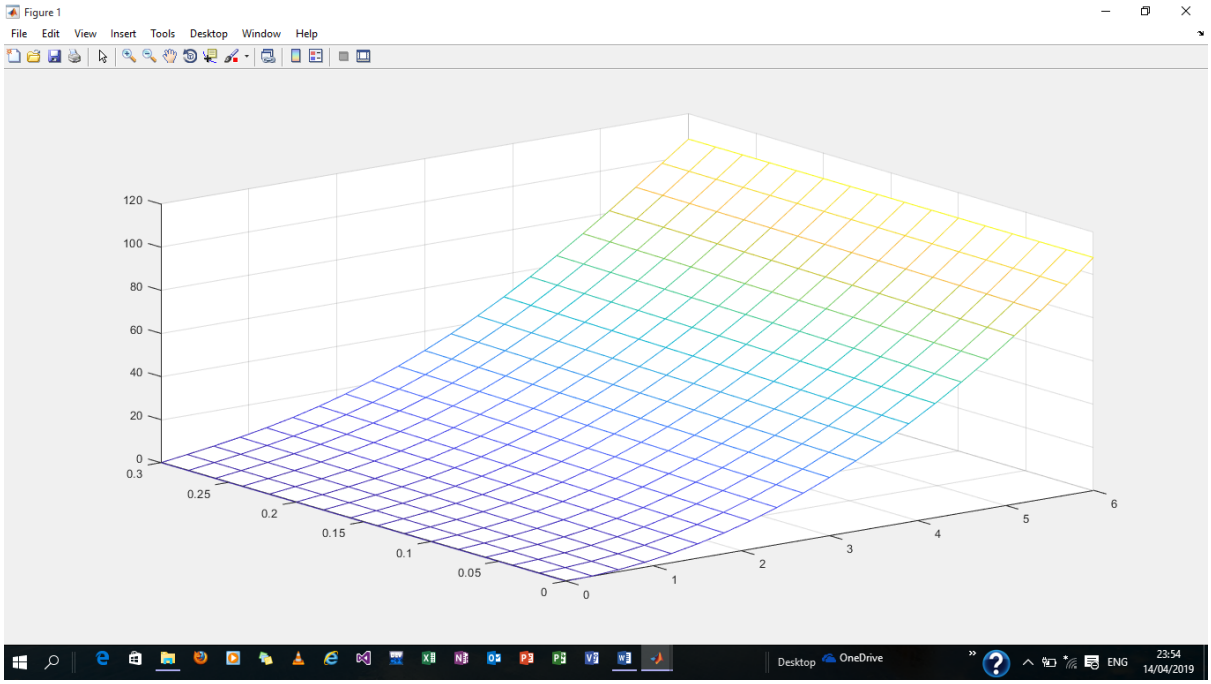


Figure 1
File Edit View Insert Tools Desktop Window Help
120
100
80
60
40
20
0
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
Desktop OneDrive
ENG
23:54
14/04/2019