

Question 1 [20 Marks]

The model for the temperature distribution in a rod of length L = 6m is as given in Equation (1),

$$\frac{\partial T(x,t)}{\partial t} = c \frac{\partial^2 T(x,t)}{\partial x^2} \dots\dots\dots(1)$$

where

$$c = 2.2 \text{ cm}^2 / \text{hr} \dots\dots\dots(2)$$

with the conditions that the temperature (°C):

$$T(x, 0) = 3x^2 \dots\dots\dots (3)$$

$$T(0, t) = 0 \dots\dots\dots(4)$$

$$T(L,t) = 108 \dots\dots\dots(5)$$

Using $\Delta t = 0.02$ hr and $\Delta x = 3.0$ cm, obtain the temperature profile of the system for $0 \leq t \leq 3.0$ hr

(a) manually, in tabular form, solving up to $t = 0.02$ hr and $x = 6$ cm

SOLUTION

$$\frac{\partial T(x,t)}{\partial t} = c \frac{\partial^2 T(x,t)}{\partial x^2}$$

Simplifying the above equation using the forward difference method for first order and the central difference for the second order; gives:

$$\frac{T_{i+1,j} - T_{i,j}}{\Delta t} = c \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{(\Delta x)^2}$$

$$T_{i+1,j} - T_{i,j} = \frac{c\Delta t}{(\Delta x)^2} (T_{i,j+1} - 2T_{i,j} + T_{i,j-1})$$

$$\text{Let } r = \frac{c\Delta t}{(\Delta x)^2}$$

$$T_{i+1,j} - T_{i,j} = r (T_{i,j+1} - 2T_{i,j} + T_{i,j-1})$$

$$T_{i+1,j} = r (T_{i,j+1} - 2T_{i,j} + T_{i,j-1}) + T_{i,j}$$

$$T_{i+1,j} = r T_{i,j+1} + (1-2r) T_{i,j} + r T_{i,j-1}$$

The boundary conditions are:

$$T(0, t) = 0$$

$$T(L,t) = 108$$

$$\text{And } L = 6\text{cm}$$

$$T(x, 0) = 3x^2$$

$$c = 2.2 \text{ cm}^2 / \text{hr}$$

$$\Delta t = 0.02 \text{ hr}$$

$$\Delta x = 0.3\text{cm}$$

$$r = \frac{c\Delta t}{(\Delta x)^2} = \frac{2.2 \cdot 0.02}{0.3^2} = 0.4889$$

$$nx = \frac{x}{\Delta x} = \frac{6}{0.3} = 20$$

$$nt = \frac{t}{\Delta t} = \frac{0.3}{0.02} = 15$$

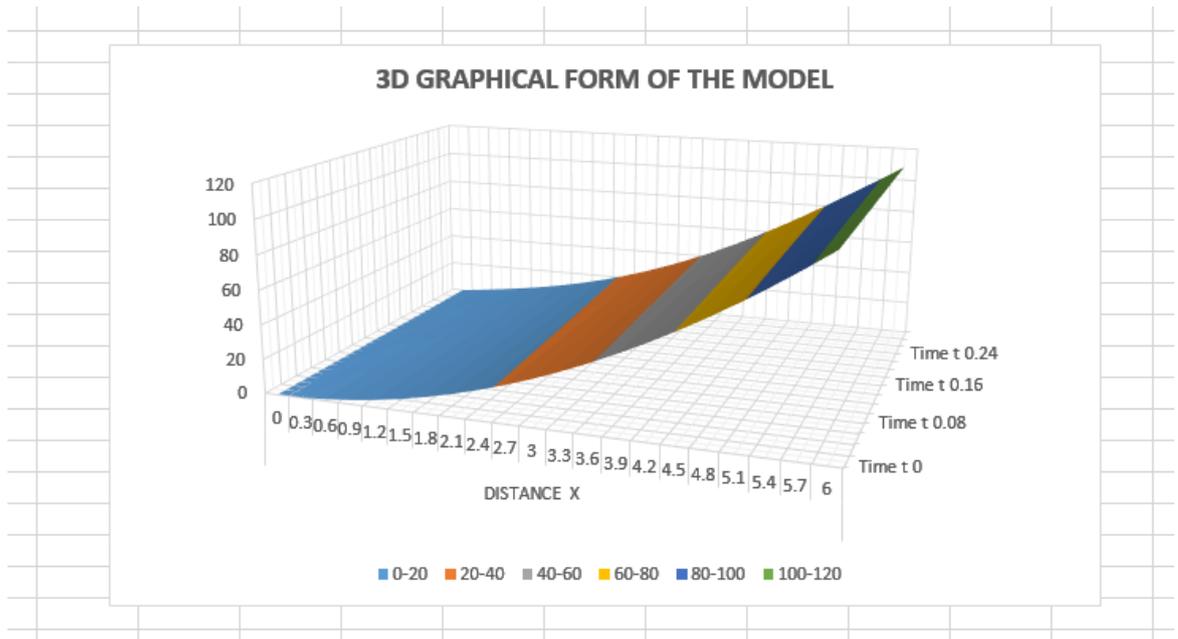
		TIME t VALUES		
Distance x values	j	i	0	1
			0.00	0.02
	0	0.0	0	0
	1	0.3	0.27	0.534

	2	0.6	1.08	1.344
	3	0.9	2.43	2.694
	4	1.2	4.32	4.584
	5	1.5	6.75	7.014
	6	1.8	9.72	9.984
	7	2.1	13.23	13.494
	8	2.4	17.28	17.544
	9	2.7	21.87	22.134
	10	3.0	27.00	27.264
	11	3.3	32.67	32.934
	12	3.6	38.88	39.144
	13	3.9	45.63	45.894
	14	4.2	52.92	53.184
	15	4.5	60.75	61.014
	16	4.8	69.12	69.384
	17	5.1	78.03	78.294
	18	5.4	87.48	87.744
	19	5.7	97.47	97.734
	20	6.0	108.00	108

(b) with the aid of Microsoft Excel, in tabular and 3D graphical forms, and
SOLUTION

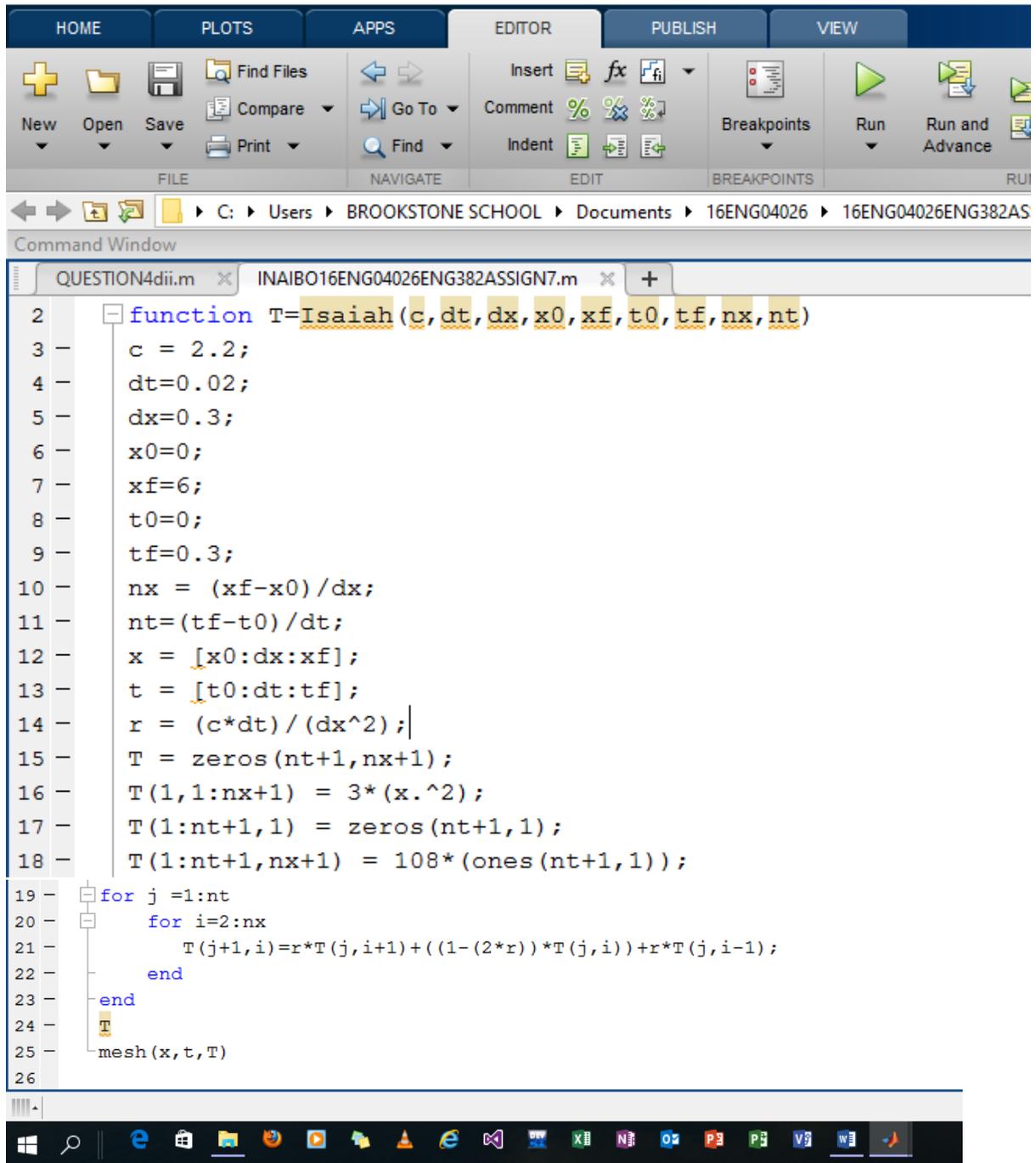
The screenshot shows an Excel spreadsheet with the following structure:

- Columns:** A through S. Column A contains labels (xf, tf, n, m, dx, dt, r, c). Column B contains numerical values. Column E is the active cell with the formula $=\$B\$7*D15+(1-2*\$B\$7)*D16+\$B\$7*D17$.
- Row 10:** Header for "Time t" with values from 0 to 0.3 in increments of 0.02.
- Row 11:** Header for "DISTANCE X" with values from 0 to 108 in increments of 6.
- Data:** A grid of numerical values representing distance at various time intervals. For example, at time 0.02, distance is 0.3; at time 0.04, distance is 0.6; at time 0.06, distance is 0.9; and so on, up to time 0.3 where distance is 108.



(c) with the aid of MATLAB, in tabular and 3D graphical forms.

SOLUTION



The image shows the MATLAB R2018a software interface. The top menu bar includes HOME, PLOTS, APPS, EDITOR, PUBLISH, and VIEW. Below the menu bar is a toolbar with icons for New, Open, Save, Find Files, Compare, Print, Go To, Find, Insert, Comment, Indent, Breakpoints, Run, and Run and Advance. The address bar shows the file path: C:\Users\BROOKSTONE SCHOOL\Documents\16ENG04026\16ENG04026ENG382AS. The Command Window is open, displaying the following MATLAB code:

```
2 function T=Isaiah(c,dt,dx,x0,xf,t0,tf,nx,nt)
3     c = 2.2;
4     dt=0.02;
5     dx=0.3;
6     x0=0;
7     xf=6;
8     t0=0;
9     tf=0.3;
10    nx = (xf-x0)/dx;
11    nt=(tf-t0)/dt;
12    x = [x0:dx:xf];
13    t = [t0:dt:tf];
14    r = (c*dt)/(dx^2);
15    T = zeros(nt+1,nx+1);
16    T(1,1:nx+1) = 3*(x.^2);
17    T(1:nt+1,1) = zeros(nt+1,1);
18    T(1:nt+1,nx+1) = 108*(ones(nt+1,1));
19    for j =1:nt
20        for i=2:nx
21            T(j+1,i)=r*T(j,i+1)+((1-(2*r))*T(j,i))+r*T(j,i-1);
22        end
23    end
24    T
25    mesh(x,t,T)
26
```

The Windows taskbar is visible at the bottom of the screen, showing various application icons.

