

(a) Mathematical Modelling can be defined as the mathematical representation of a system and simulation of a system involving the model and obtaining its mathematical solution.

- (b)
- By the use of balance law
 - By differential equation
 - By use of newtons law of cooling.

(c) $\frac{dT}{dt} \propto (T - T_A)$

$$\frac{dT}{dt} = k(T - T_A)$$

$$\frac{1}{T - T_A} dT = k dt$$

$$\ln(T - T_A) = kt + c$$

$$T - T_A = e^{kt + c}$$

$$T - T_A = e^{kt} \cdot e^c$$

$$T - T_A = T_0 e^{kt}$$

@ $t = 0$ $T = 25$

$$T - T_A = T_0$$

$$T - 25 = T_0$$

$$10 - 25 = T_0$$

$$\therefore T_0 = -15$$

$$T - 25 = -15e^{kt}$$

$$T = 25 - 15e^{kt}$$

@ $t = 5 \text{ min}$, $T = 20$

$$20 - 25 = -15e^{k(5)}$$

$$-5 = -15e^{5k}$$

$$e^{5k} = \frac{-5}{-15} = \frac{1}{3}$$

$$5R = \ln\left(\frac{1}{3}\right)$$

$$k = \ln\left(\frac{1}{3}\right) / 5$$

$$k = -0.22$$

$$T = 25 - 15e^{-0.22t}$$

④ MATLAB

→ Command window

clc

clear

$$T = 25 - (15 * \exp(-0.22 * t))$$

$$t = 0:0.5:50$$

$$T_n = \text{subs}(T)$$

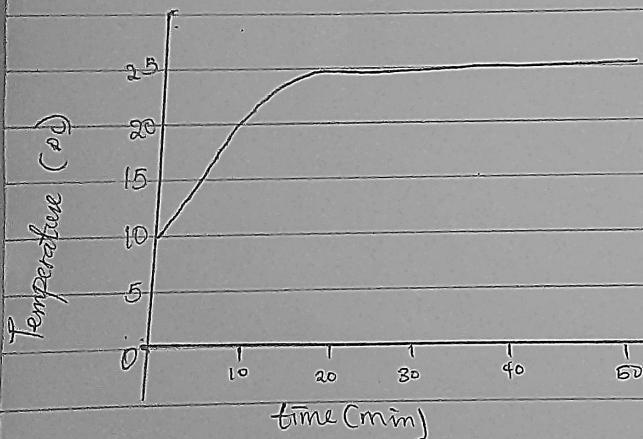
Plot (t, T_n)

grid on

grid minor

xlabel('time(min)')

ylabel('Temperature(°C)')



② the steady-state of the system is 25°C.

③ The system was steady after 20 min

④ at $T = 24.9^\circ\text{C}$

$$T = 25 - 15e^{-0.22t}$$

$$24.9 - 25 = -15e^{-0.22t}$$

$$\frac{-0.1}{-15} = e^{-0.22t}$$

$$-15$$

$$e^{-0.22t} = 0.0067$$

$$0.22t = \ln(0.0067)$$

$$t = \frac{\ln(0.0067)}{-0.22}$$

$$t = 22.76 \text{ mins}$$

~~$$t = 22.76 \text{ seconds}$$~~

$$t = 22 \text{ min, } 76 \text{ seconds}$$